DEVELOPMENT OF A SIMPLIFIED CAT ALGORITHM, INVOLVING BESSEL FUNCTIONS

A Thesis Submitted
in Partial Fulfilment of the Requirements
for the Degree of

MASTER OF TECHNOLOGY

by

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to the

NUCLEAR ENGINEERING & TECHNOLOGY PROGRAMME

INDIAN INSTITUTE OF TECHNOLOGY KANPUR

MAY, 1987

DEDICATED

TO

MY PARENTS

CERTIFICATE

This is to certify that this work on "Development of a Simplified CAT Algorithm, involving Filtered Bessel Functions" by Major V.K. Bhatia has been carried out under our supervision and has not been submitted elsewhere for the award of a degree.

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ACKNOWLEDGEMENTS

It gives me immense pleasure to take this opportunity to express my gratitude with reverence to my thesis supervisors Mr. Prabhat Munshi and Dr. R.K.S. Rathore, who initiated me into the work reported in this thesis and provided with invaluable and unsevered guidance for carrying out this work. Without their help and encouragement, this work would never have reached a completion.

I also gratefully acknowledge the invaluable advice provided by Professor K. Sri Ram.

I am highly indebted for the regular help provided by Mr. S. Pandimani, Miss Nivedita Patel, Major S.C. Dey and all my colleagues, along with their co-memorable company, which will be cherished throughout the course of life.

I would also like to express my gratitude to the cohesive and well-knit NET family of faculty members, the staff and the students for their regular encouragement provided all along my stay.

The credit also goes to the lady behind a man (for his successes), my wife Reva not forgetting Vaishali for their regular moral support.

-Major V.K. Bhatia

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ABSTRACT

Computer Aided Tomography (CAT) is widely used in the medical imaging area for the detection of cancerous tissues. It has also proved to be a good technique for measuring point-density (void-fraction) in two-phase flow systems. The present day CAT-scanners employ algorithms which are essentially a discrete implementation of the Radon Inversion Formulae. These methods of reconstructing density distributions are fast and easy to implement.

The present work is an attempt to simplify (and consequently to speed-up) the existing algorithms for the cases of radially symmetric density distribution. The method uses Radon Inversion Formulae and involves Bessel's Function of Fourier frequency.

The algorithm was tested on some known distributions and the reconstruction is excellent. Data for bubbly-flow was also processed to obtain a point density distribution. The range of investigation was $0.6 \leq \langle\rho\rangle \leq 0.9$ gm/cm^3, where " $\langle\ \rangle$ " denotes average cross-sectional value. The values of $\langle\rho_{CT}\rangle$ are within $\pm$ 0.03 g/cm^3 of the true values, $\langle\rho\rangle$, for the data sets considered. The results are comparable with another simple method involving radial polynomials. It is observed that the simplified Radon Inversion Formulae for the radially symmetric case gives good results and can be implemented with ease.

CHAPTER 1

INTRODUCTION

The problem of image reconstruction from projections has repeatedly arisen over the last four decades in a large number of scientific, medical and technical fields. Of all the applications, probably the greatest effect on the world at large has been in the area of diagnostic medicine. Computer-Aided-Tomography (CAT) has revolutionized the techniques in radiology.

Images of cross-sections of the human body are produced from the data obtained by measuring the attenuation of radiation along a large number of lines through the cross-section. The CAT machine incorporates scanning of patient with radiation using appropriate tomographic algorithm to reconstruct the density distribution of tissues [1].

A major problem confronting an engineer in the Nuclear reactor safety field is the measurement of void fraction (or density) during various accident/transient situations. The development of an accurate and reliable two-phase flow measurement technique for various flow systems and components in Light Water Reactor (LWR) is of great importance and significance. The concept of measuring density distribution was first investigated by Schlosser et al [2] for a two-phase air-water system. The density measurement

results have been summarized by Kulacki et al [3].

In many applications where images are required, the measurements can be done by probing the object by invisible radiations and interpreting the measured data. Sometimes immediate interpretation of measured data is not possible directly, but is related to the image in a known way. Generally, all image reconstruction procedures or algorithms aim at processing the data to form the image and thereby to facilitate the interpretation of measurement.

The present work mainly looks at such a new field, which will alleviate the problems (related to radially symmetric distribution) in the nuclear engineering area (two-phase flow in reactor systems), chemical industry, food processing etc.

The most important feature of tomographic reconstruction method is its non-invasive aspect of point-density measurement. Due to this fact, the system retains its natural/original configuration, without being even slightly affected by the measuring device.

The present work is aimed at developing a simplified algorithm based on the Radon Inversion Formulae,involving Bessel Functions, which can perform tomographic reconstruction of the density distribution for radially symmetric two-phase flows. The data is collected in the parallel-beam mode.

The algorithm written in FORTRAN, has been initially tested against some known functions, simulated radially symmetric distributions, and annular flows. The algorithm has also been successfully implemented to reconstruct the density map for air-water bubbly flow data [3,4] . The results are comparable with the results obtained by the other known general methods [3,4,5] . The results are also comparable with another alternate scheme involving radial polynomials [6,7].

CHAPTER 2

THEORETICAL FORMULATION

This chapter includes a brief discussion of:

(a) methods of data collection in a tomographic system depending upon geometry and,

(b) the relation of this data to the distribution.

Two standard geometries of data collection, namely, fan beam and parallel beam geometries are discussed. In addition, types of algorithms, two phase flow and use of CAT in identifying flow regimes are briefly explained.

2.1 Data Collection

In a tomographic system, radiation is passed through the object (to be reconstructed) and the attenuated signal is detected. This signal is related to the function to be reconstructed. The single beam radiation attenuation phenomenon is represented by:

$$N = N_o \exp\{- \int_c \mu(r,\emptyset) ds\} \tag{1}$$

where,

N = detector reading (counts/s)

N_o = source strength (counts/s)

S = path of radiation (ray)

C = chord along which S is integrated

μ = absorption coefficient

$r,\emptyset$ = cylindrical co-ordinates.

Rewriting eqn.(1), we have

$$p = \int_C \mu(r,\emptyset)ds \tag{2}$$

where

$$p = \ln (N_o/N) \tag{3}$$

2.2 Parallel Beam Geometry Data Collection

A line in the plane is specified by two parameters; its (signed) distance l from the origin and its angle θ with respect to the Y-axis (Fig.1). The function of two variables, whose value for any (l,θ) is defined as line-integral of μ along the line specified by l and θ, is denoted by $p(l,\theta)$. For any l and θ, (l,θ) and $(-l, \theta + \pi)$ represents the same line in the plane, so that

$$p(l,\theta) = p(-l, \theta + \pi) \tag{4}$$

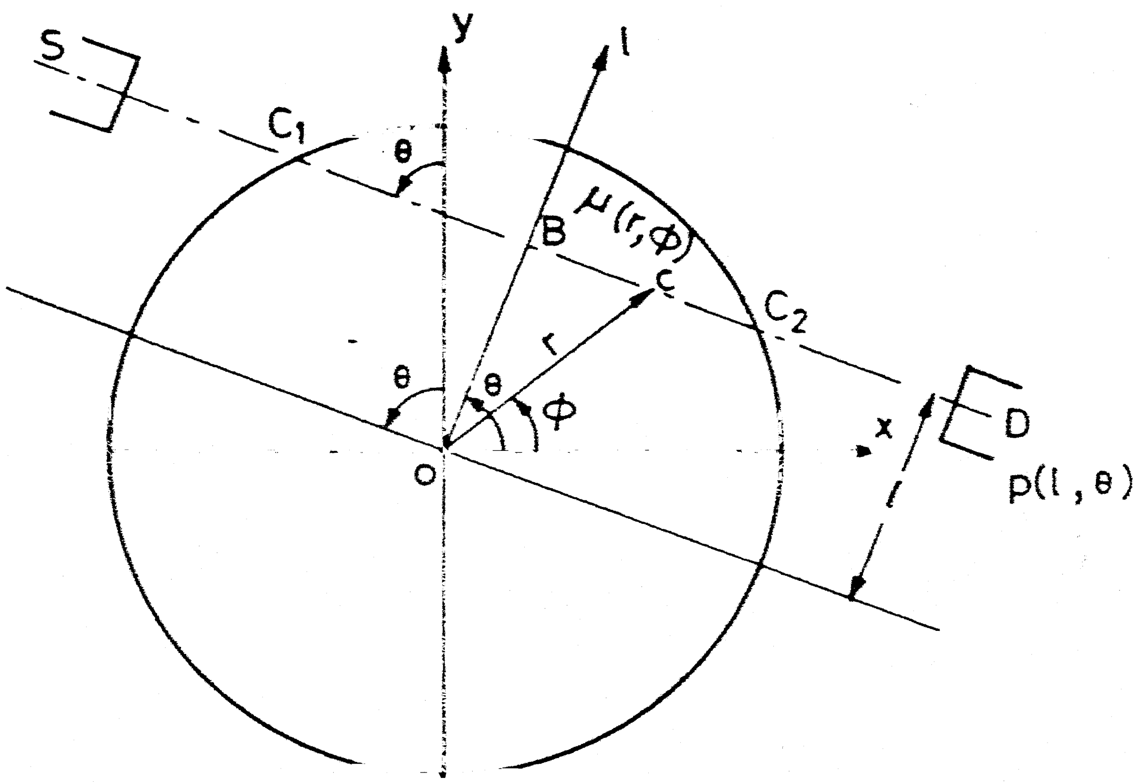
By projection theorem [8] we know that

$$\hat{\mu}(R \cos\theta, R \sin\theta) = \hat{p}(R,\theta) \tag{5}$$

where

$$\hat{p}(R,\theta) = \int_{-D}^{D} p(l,\theta)\, e^{-i2Rl\pi}\, dl \tag{6}$$

PARALLEL BEAM GEOMETRY



LEGEND: S - Source

D - Detector

Points within object are described by a fixed (x, y) or (r, Φ) co-ordinate system. Rays (dashed-line) are specified by their angle with respect to y axis, θ and their distance from origin l.

Fig.1 Data collection geometry

where

D is the object radius and

R corresponds to the Fourier frequency.

The Radon Inversion Formulae [1] relates the data $p(l,\theta)$ to the function $\mu(r,\emptyset)$ as follows:

$$\mu(r,\emptyset) = \int_{0}^{\pi} \int_{-\infty}^{\infty} \hat{p}(R,\theta)\ e^{i2\pi R \cos(\theta-\emptyset)}\ |R|\,dR\ d\theta \qquad (7)$$

Considering the radially symmetric flow into account; μ no longer remains a function dependent on $\emptyset$, so that we can choose any particular $\emptyset$ and obtain the reconstructed $\mu(r)$. For computational ease, and without loss of generality, we choose $\emptyset = 0$. Thus eqn.(7) becomes,

$$\mu(r) = \int_{0}^{\pi} \int_{-\infty}^{\infty} \hat{p}(R)\ e^{i2\pi R \cos\theta}\ |R|\,dR\ d\theta \qquad (8)$$

where

$$\hat{p}(R) = \int_{-D}^{D} p(l)\ e^{-i2\pi R l}\ dl \qquad (9)$$

We see that $p(l,\theta)$ has been replaced by $p(l)$ since radial symmetry in μ implies the independence of p with respect to the source position, θ.

Interchanging order of integration in eqn.(8), we get

$$\mu(r) = \int_{-\infty}^{\infty} \hat{p}(R)\ \left(\int_{0}^{\pi} e^{i2 R\pi \cos\theta}\ d\theta \right) |R|\ dR$$

Here integral over θ is a Bessel Function [9], so that the inversion formula takes a particularly simple form,

$$\mu(r) = 2\pi \int_0^{\infty} \hat{p}(R)\, J_o(2\pi rR)|R|\, dR \qquad (10)$$

A practical implementation would require a finite cut-off for the variable, R, the Fourier frequency. Denoting this cut-off by R_c, we get

$$\mu(r) \simeq 2\pi \int_0^{R_c} \hat{p}(R)\, J_o\, (2\pi rR)\, |R|\, dR \qquad (11)$$

Keeping in mind the discrete nature of the data p(l), a reasonable choice of R_c, based on sampling theorem [10], is given by

$$R_c = 1/2\Delta l$$

where, Δl is spacing between the rays. It is to be noted that the expression given in eqn.(11) is similar to the point-spread function expression reported by Lewitt [8].

In the design of reconstruction algorithm using Transform approach, it is often necessary to introduce a window function W(R) :

$$W(R) = \begin{cases} W(R) & |R| < R_c \\ 0 & |R| \geq R_c \end{cases}$$

giving eqn.(11), the following form:

$$\mu(r) \simeq 2\pi \int_{o}^{R_c} \hat{p}(R)\, J_o(2\pi rR)\, |R|\, W(R)\, dR \qquad (12)$$

Two different types of window functions have been tested, viz., linear and sinc functions. A sensitivity study has also been performed on the slope of the linear window function. The forms used are:

Linear

$$W(R) = 1 - \frac{\varepsilon R}{R_c}$$

where

$$0 \leq \varepsilon \leq 1.0$$

Sinc

$$W(R) = \frac{\mathrm{Sin}(\pi R/R_c)}{(\pi R/R_c)}$$

2.3 Fan Beam Geometry

In this case, one needs a rotating source and a ring of stationary detectors on the outer periphery. Scanning involves only motion of the source.

In medical applications of CT, in order to reduce distortion due to movement of patient, data should be collected in shortest possible time. This is facilitated by

using fan beam geometry of Radiation diverging from a source, passing through the patient and intercepted by detector elements. From the transmitted radiation, intensities are recorded while source moves in a circular path around the patient.

The relationship between both the geometries [8], is as follows:

$$l = D \ \mathrm{Sin}\,\sigma \ , \qquad \theta = \beta + \sigma$$

$$p(l,\theta) = g(\sigma,\beta) \ .$$

2.4 Types of Algorithms

For image reconstruction from projection data, the algorithms may be categorised as under:

(a) Transform Methods [8]

(b) Series Expansion Method [11] and

(c) Distributional Approximation Method .

(a) Transform Methods:

Based on Radon Inversion Technique, the Transform methods basically rely upon analytic formulae . Transform methods [8] are broadly categorized as under:

(i) Direct Fourier Inversion (DFI): In this method, direct Fourier Transform of projection data is taken. Subsequently 2-d Fourier Inversion

leads to the reconstruction of unknown distributions.

(ii) Convolution Back Projection (CBP): This method is based on convolving data with a suitable filter function. The back projection of this convolved data results in the reconstruction of unknown distribution.

(b) Series Expansion Methods:

In this method pixel-wise distribution of the function under-investigation is assumed. To obtain reconstruction of function in the region of interest [11], suitable iterative and non-iterative methods are applied.

The iterative SEMs are:

(i) Algebraic Reconstruction Technique and
(ii) Simultaneous Iterative Reconstruction Technique etc.

Whereas non-iterative SEMs are:

(i) Angular Harmonic Decomposition and
(ii) Polynomial Decomposition.

(c) Distributional Approximation Method:

The distributional approximation methods developed at IIT Kanpur involve assuming the distribution on the entire region of interest.

The RAP method [6,7] incorporates radial polynomials as the basis functions while the CSI Method [12] involves constant ring-values as the basis functions in its formulation.

The approach used in the present study involves the following steps:

(1) Read data p in the form of a column vector corresponding to the spacing between rays.

(2) Computation of $\hat{p}(R)$.

(3) Computation of μ -values along the chord using eqns. (11,12) (involving Bessel Function).

(4) Calibration of μ -values to the density values.

For the validation studies, data was generated (eqn.(2)), whereas in the experimental air-water flow studies, data is available from the detector counts. For computation of $\hat{p}(R)$, a simple integration scheme was adopted (Appendix E).

2.5 Two-Phase Flow

Some of the major problems in the nuclear industry are:

(a) Measurement of mass-flow rate of two-phase flow,

(b) Finding nature of flow-around fuel bundle in Light Water Reactor (LWR),

(c) Developing reliable two-phase flow measuring technique for various flow systems in a reactor.

Earlier photographic methods for identification of flow regimes, using appropriate correlation to measure void-fraction were used. With the development of CAT, problem of determination of mass flow rate and identification of flow regimes has been simplified.

Different two-phase flow regimes, namely, bubbly, slug, annular and dispersed flows can be investigated for determining void-fractions. However, to determine mass-flow rate, it is essential to know void-fraction without disturbing the flow field. This can be achieved by Tomographic Methods much more accurately than by other existing methods.

CHAPTER 3

RESULTS

3.1 Validation Against Simulated Data

We discuss the results for the simulated data in this section. We assume the object size/pipe radius to be one unit and the ray spacing 0.1.

The algorithm was tested against the simulated data on the following symmetric distributions:

$$\mu(r) = 1.0$$

$$\mu(r) = r$$

$$\mu(r) = \exp(r) \text{ and}$$

$$\mu(r) = \exp(-r)$$

The reconstruction is within $\pm$ 2% for all the test functions except in the peripheral region. The true and reconstructed values are shown in Fig.2 .

The algorithm has also been tested for annular flows, having distributions as under:

$$\mu(r) = \left.\begin{array}{ll} 1.0 & 0 \leq r < 0.4 \\ = 0.0 & 0.4 \leq r \leq 1.0 \end{array}\right\}$$

$$\mu(r) = \left.\begin{array}{ll} 0.0 & 0 \leq r < 0.2 \\ 1.0 & 0.2 \leq r \leq 0.4 \\ 0.0 & 0.4 \leq r \leq 1.0 \end{array}\right\}$$

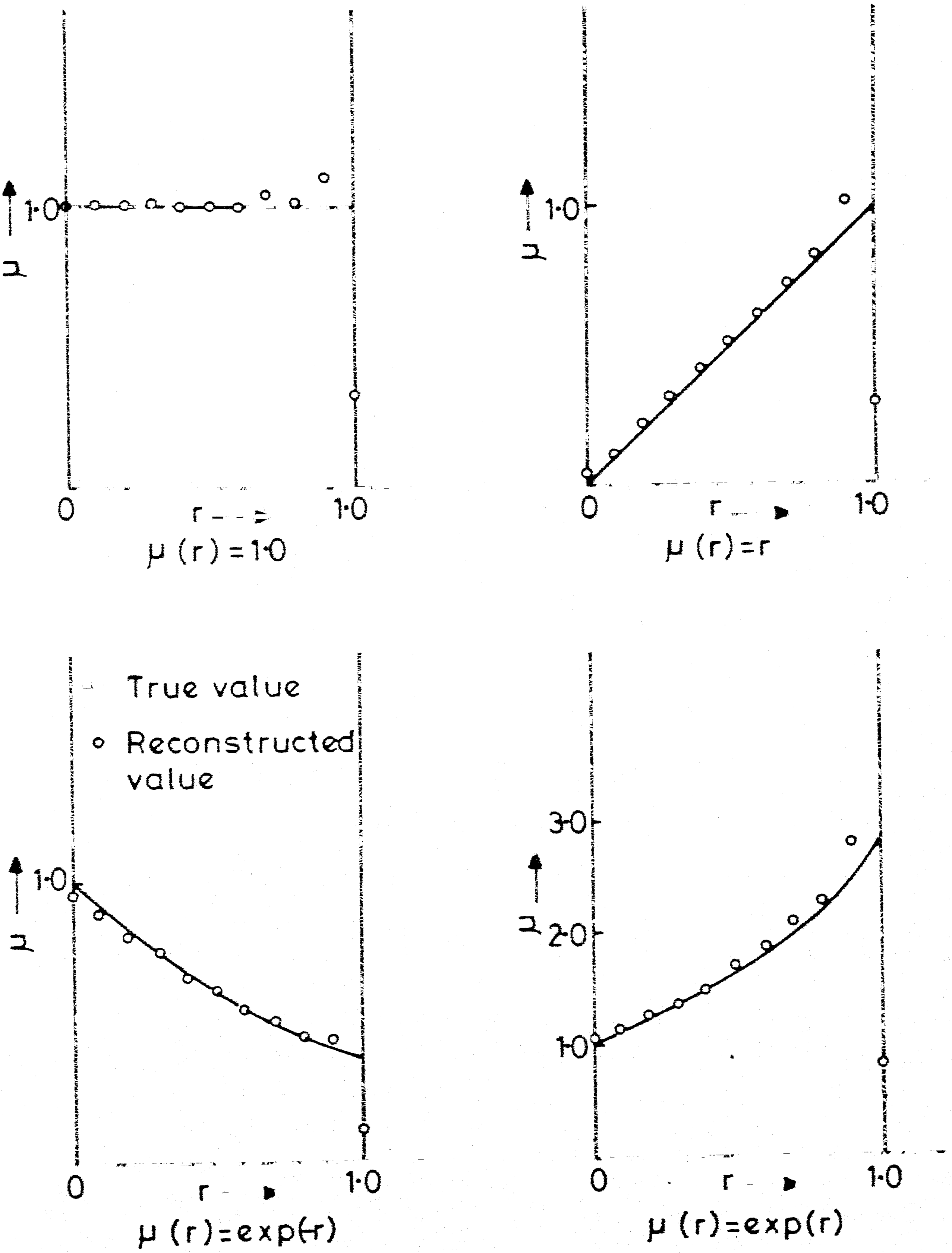


Fig.2 Reconstructed results for test-functions

In a caliberated sense, the above mentioned distributions represent the varioos types of density/void fraction distributions encountered in radially symmetric bubbly and annular flows. The reconstruction is shown in Figs.3 and 4. The solid lines represent the assumed functions and the dotted lines indicate the reconstructed values for the unfiltered and filtered versions.

Testing of algorithm for Dirac Delta (δ) function (at the center of the pipe) was also carried out. Both the filters viz. linear and sinc were used for δ-function. The results are encouraging, both for unfiltered and filtered versions (See Fig. 5).

3.2 Results for Bubbly Air Water-Flows

The test data is taken from the study of Ref. [3,4]. Five different data sets for four different densities/void fractions were processed by this algorithm. The algorithm output, CTN, was calibrated to obtain the density. To do this, the data was processed for the following four known densities:

Air, $\langle\rho\rangle = 0.00$ g/cm^3,

Pine, $\langle\rho\rangle = 0.41$ g/cm^3,

Walnut, $\langle\rho\rangle = 0.732$ g/cm^3,

Water, $\langle\rho\rangle = 1.00$ g/cm^3,

where " < > " denotes the cross-sectionally averaged values.

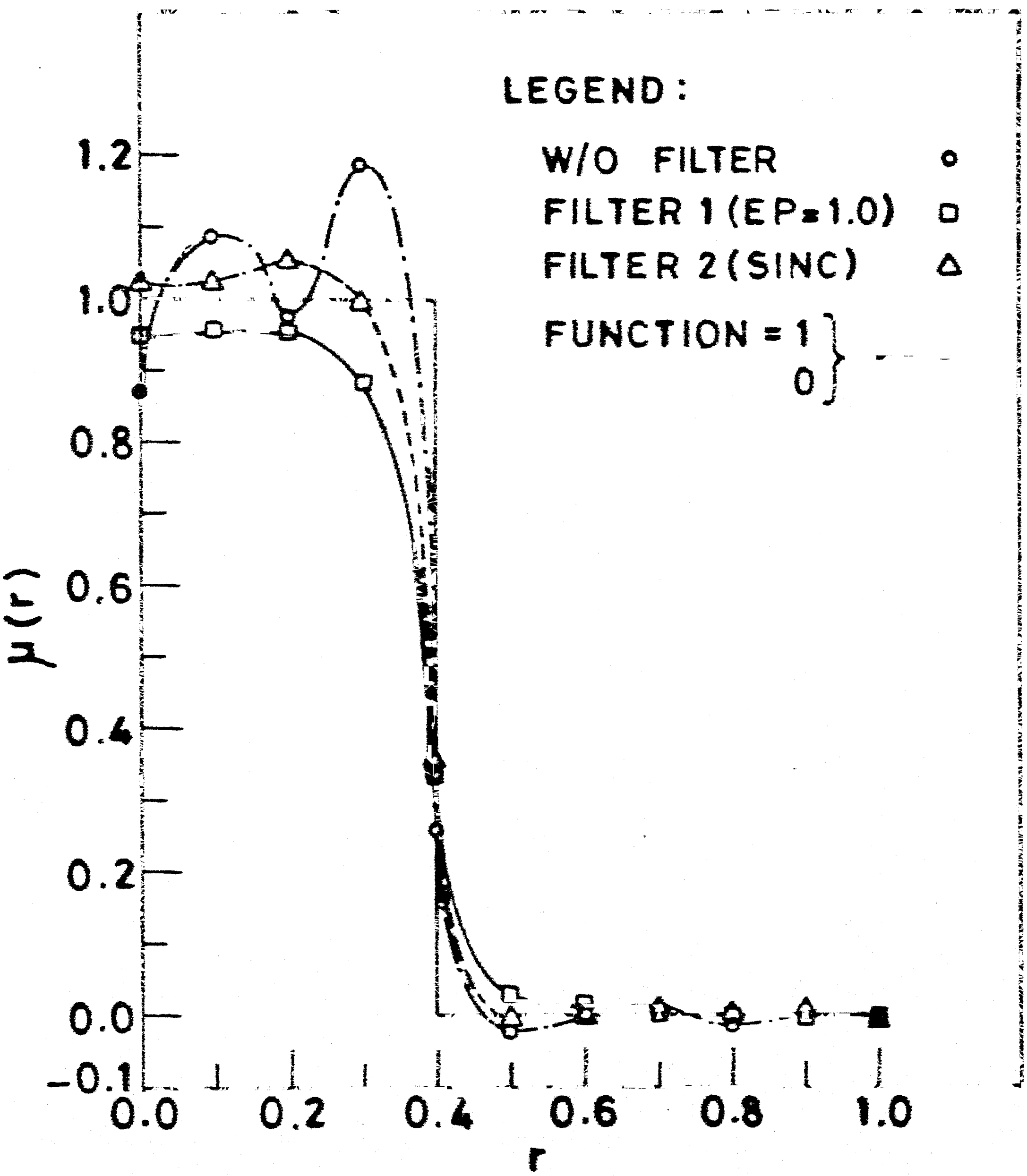


FIG. 3 RESULTS WITH FILTERED VERSION (ANNULAR FLOW).

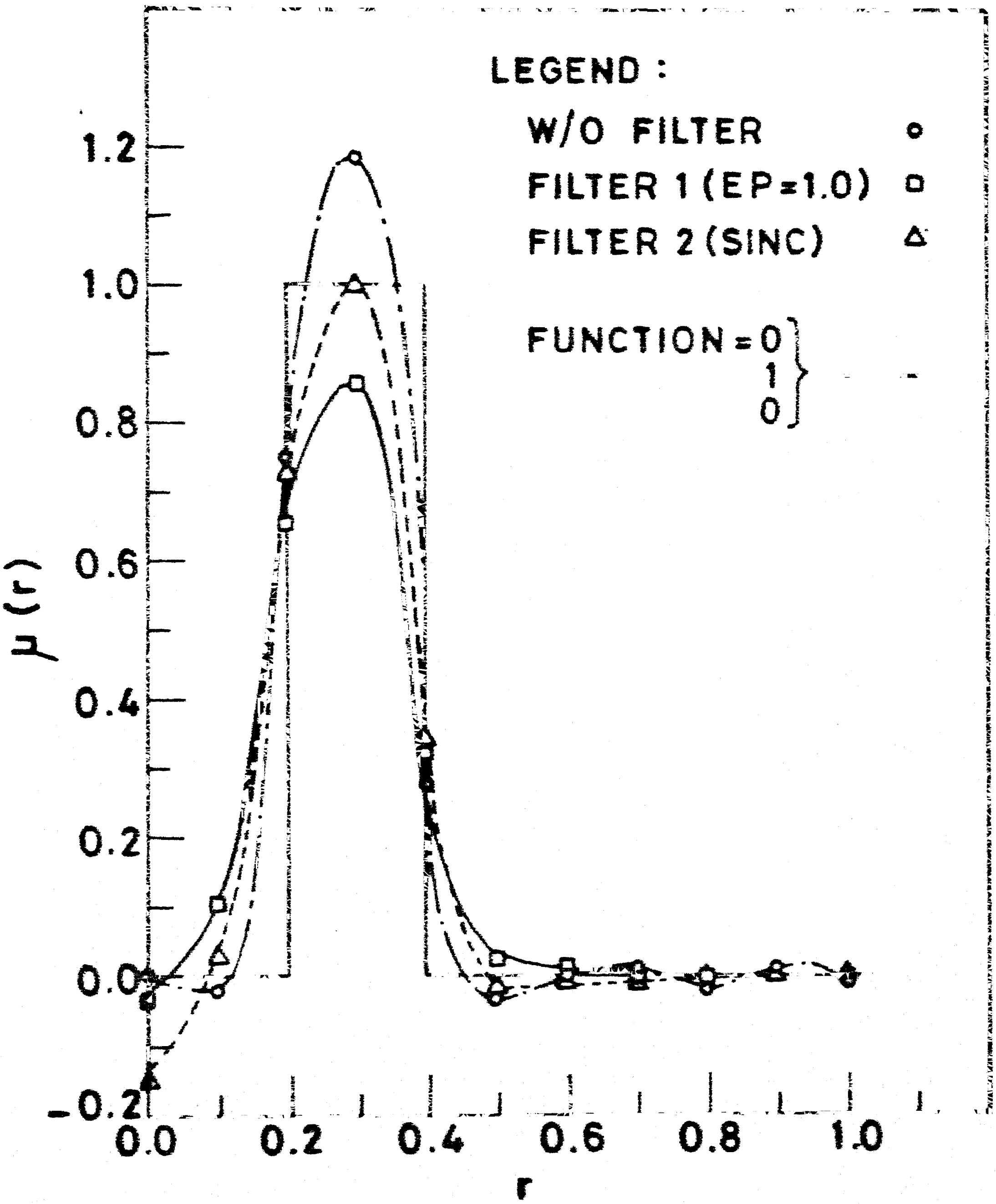


FIG. 4 RESULTS WITH FILTERED VERSION (ANNULAR FLOW).

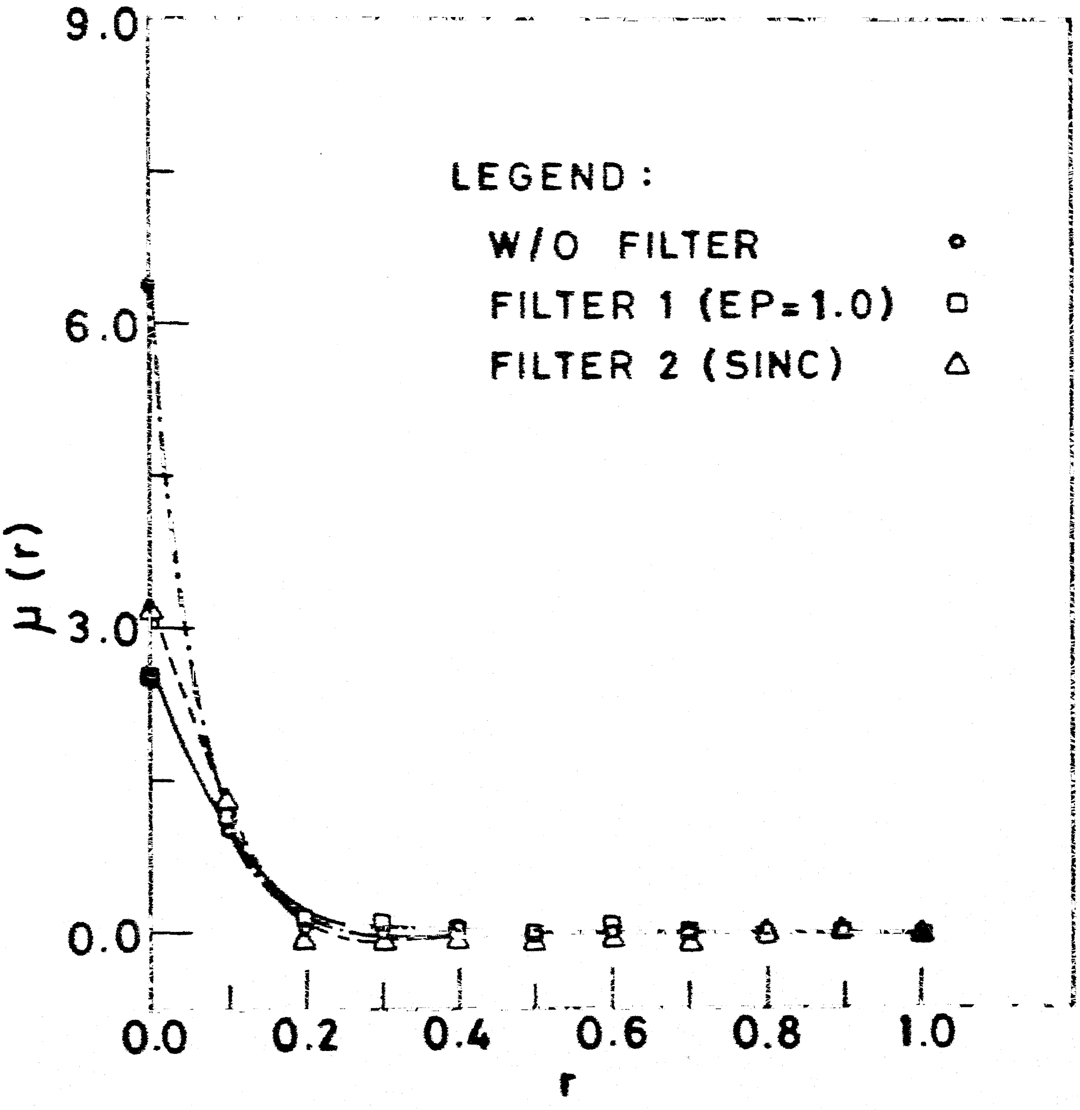


FIG.5 RESULTS WITH FILTERED VERSION (DELTA FUNCTION).

The output CTN, was corrected to the base of air (CTN = 0.0) to eliminate the effect of the pipe-wall, made of plexiglass. We know, for air, the absorption coefficient (CTN) must be zero, whereas for air data set, we get non-zero values for the <CTNs>. This effect is due to attenuation in the plexiglass pipe. In order to get the correct CTNs for all cases, we subtract the CTNs for the air (ρ = 0.0 g/cm^3) case from the reconstructed values. This way, we can get correct CTNs and <CTNs> for any case. The CTNs obtained are listed in Appendix B. The calibration curve [$\langle\rho\rangle$ vs. <CTN>] is shown in Fig. 6, by which, for any known <CTN>, the corresponding density value can be obtained.

Similarly data for different scans were processed and values of CTNs for the 12 rings in the pipe cross-section were achieved. For each radius, the ρ value obtained after calibration, have been plotted. Figures 7-11 show the reconstructed density profiles for various densities, $\langle\rho\rangle$, (or void-fractions $\langle\alpha\rangle$) for all the scans.

In Fig. 12, the comparison of reconstructed densities with actual densities (measured by an alternate method [3,4]) have been shown. The various densities obtained by calibration method are marked to show the deviation of the reconstructed densities from the actual densities. The summary of <CTN> and $\langle\rho\rangle$ for all scans is given in

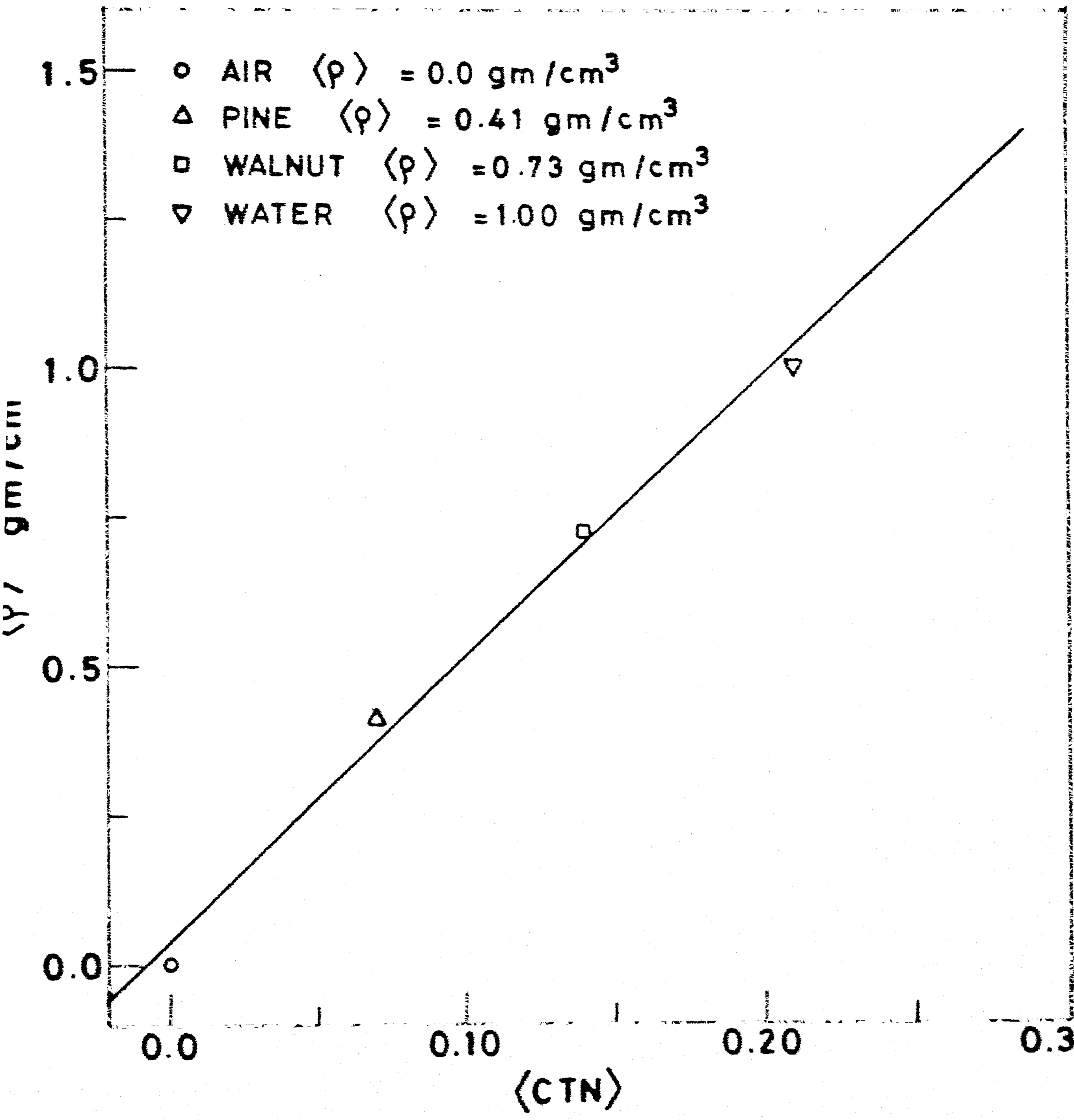


FIG. 6 CALIBRATION CURVE

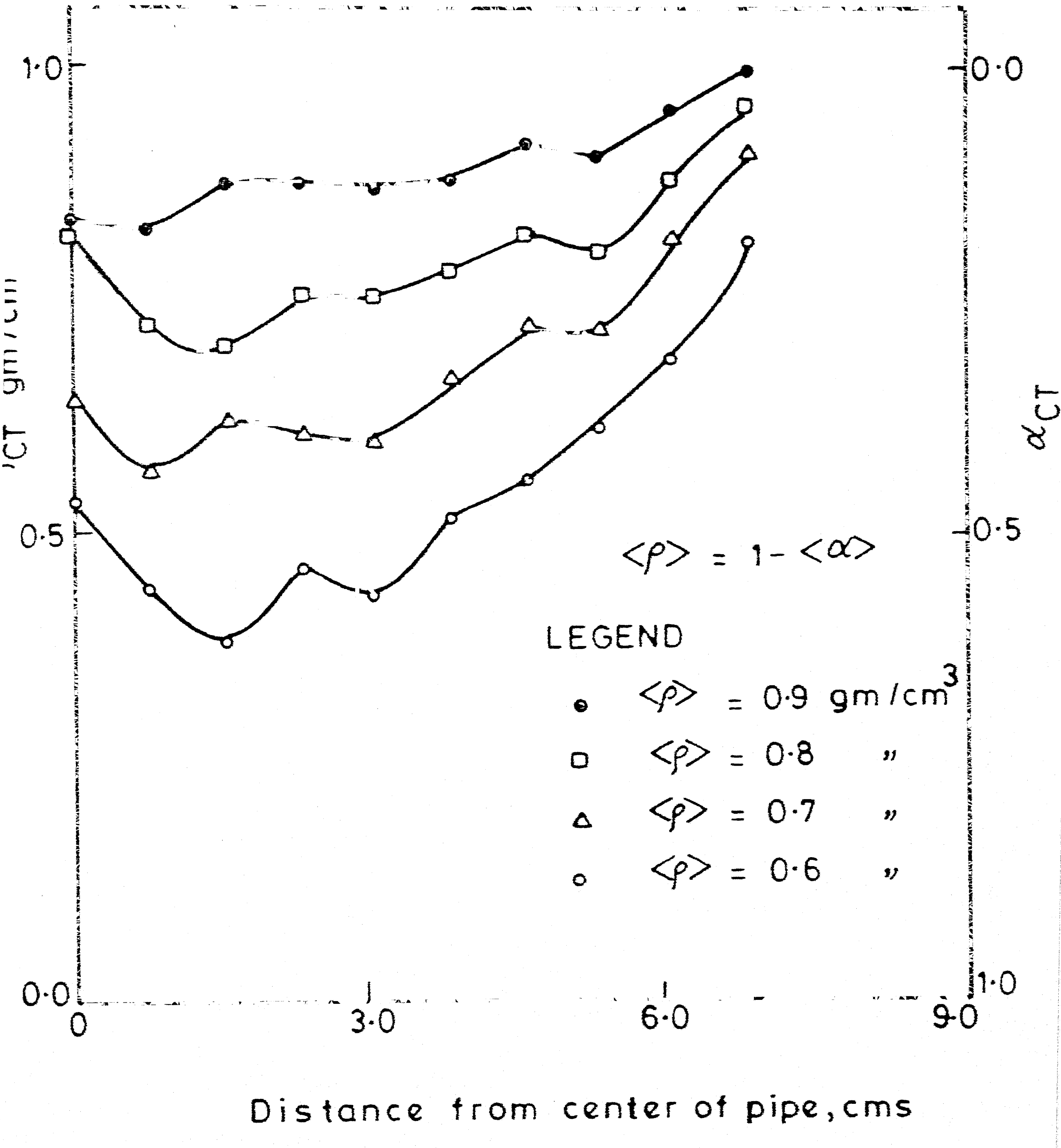


Fig. 7 Reconstructed density profile(Radial) SCAN 1.

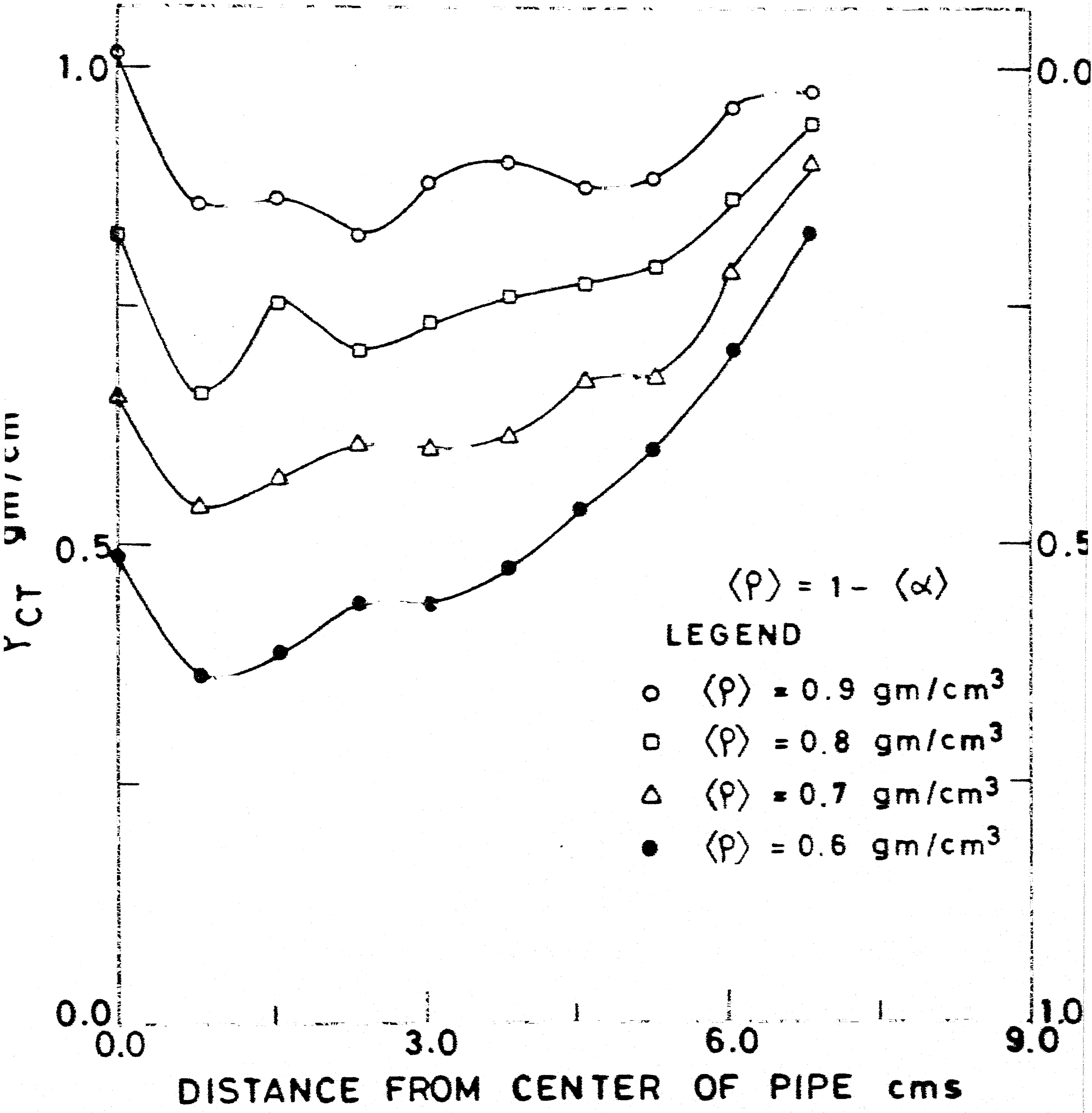


FIG. 8 RECONSTRUCTED DENSITY PROFILE (RAD
SCAN 2.

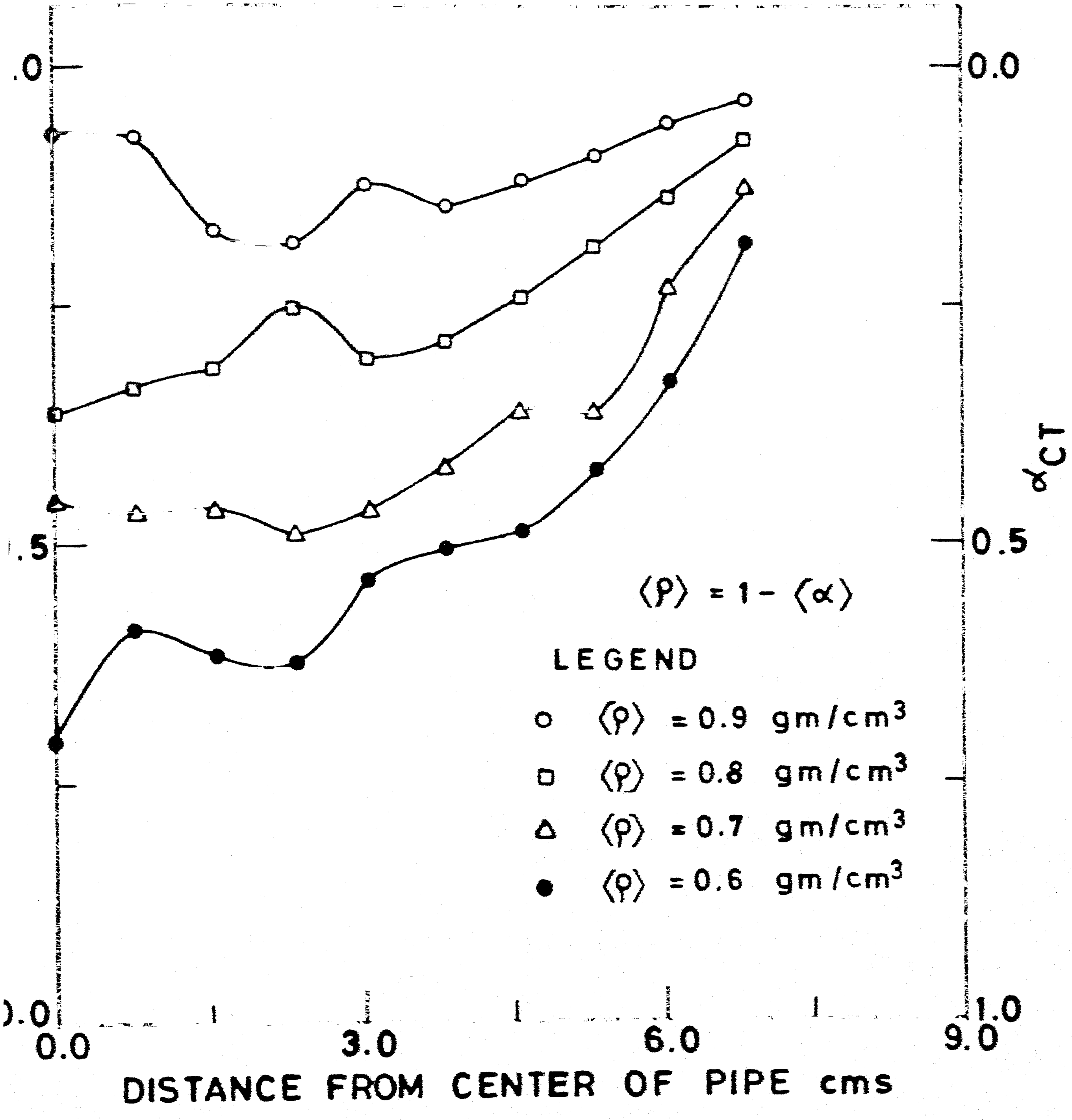


FIG. 9 RECONSTRUCTED DENSITY PROFILE (RADIAL) SCAN 3 .

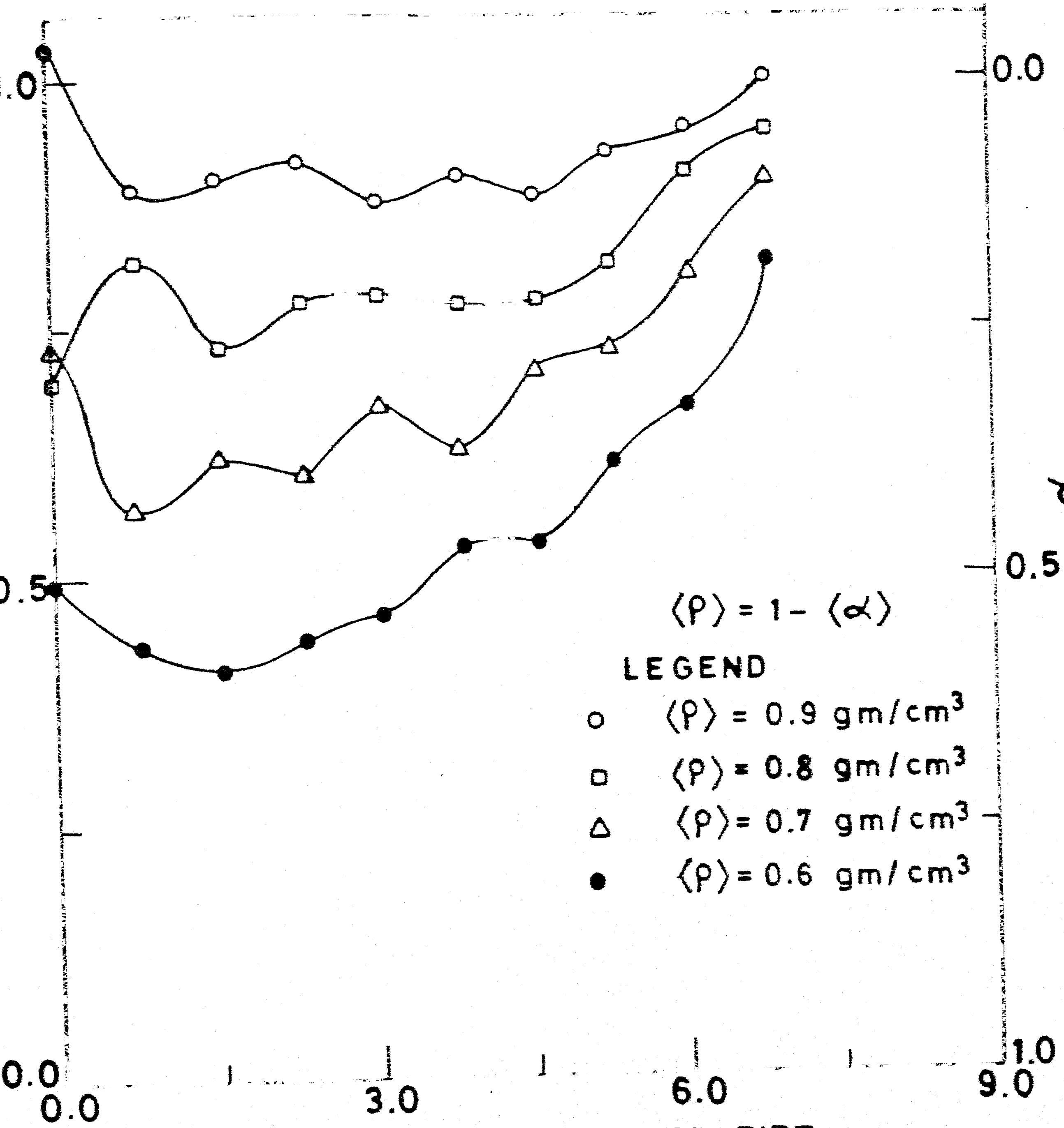


FIG.11 RECONSTRUCTED DENSITY PROFILE (RADIAL) SCAN 5.

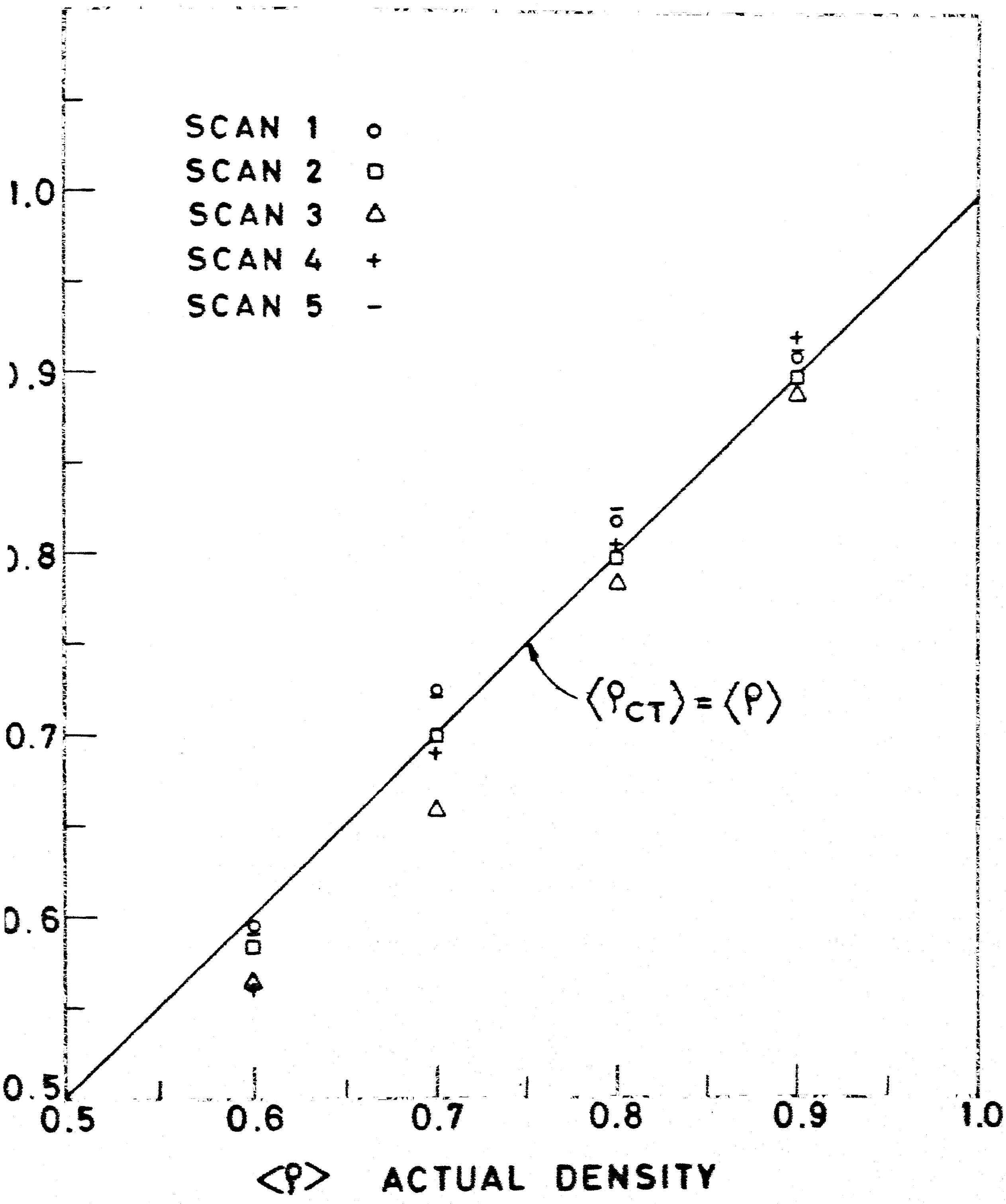


FIG.12 COMPARISON OF RECONSTRUCTED DENSITY WITH AN ALTERNATE METHOD.

Appendix B.

It is observed that the reconstruction is within $\pm$ 0.03 g/cm^3 of the actual values. The variation of error with density has been plotted in Fig.13. The variation of relative error with respect to density is shown in Fig. 14 and it ranges between + 3.57% to -6.67%.

3.3 Results with Filtered Version:

The two filters viz. linear and sinc, applied to four known cases of distributions of 1, r, exp(r) and exp(-r), give encouraging results. Figures 15-18 show the distributions without filter and with these two filters separately, for these four known cases. The deviation of the reconstructed values from the true values is within $\pm$ 0.03g/cm^3.(except for the peripheral region) for all the filters tested in the study. It is to be noted that near the edges the density falls off sharply and a reconstruction incorporating a continuous function (Bessel function in this case) naturally leads to discrepancies in the peripheral region. However such errors do not significantly effect the reconstruction quality of the overall cross-sectional image.

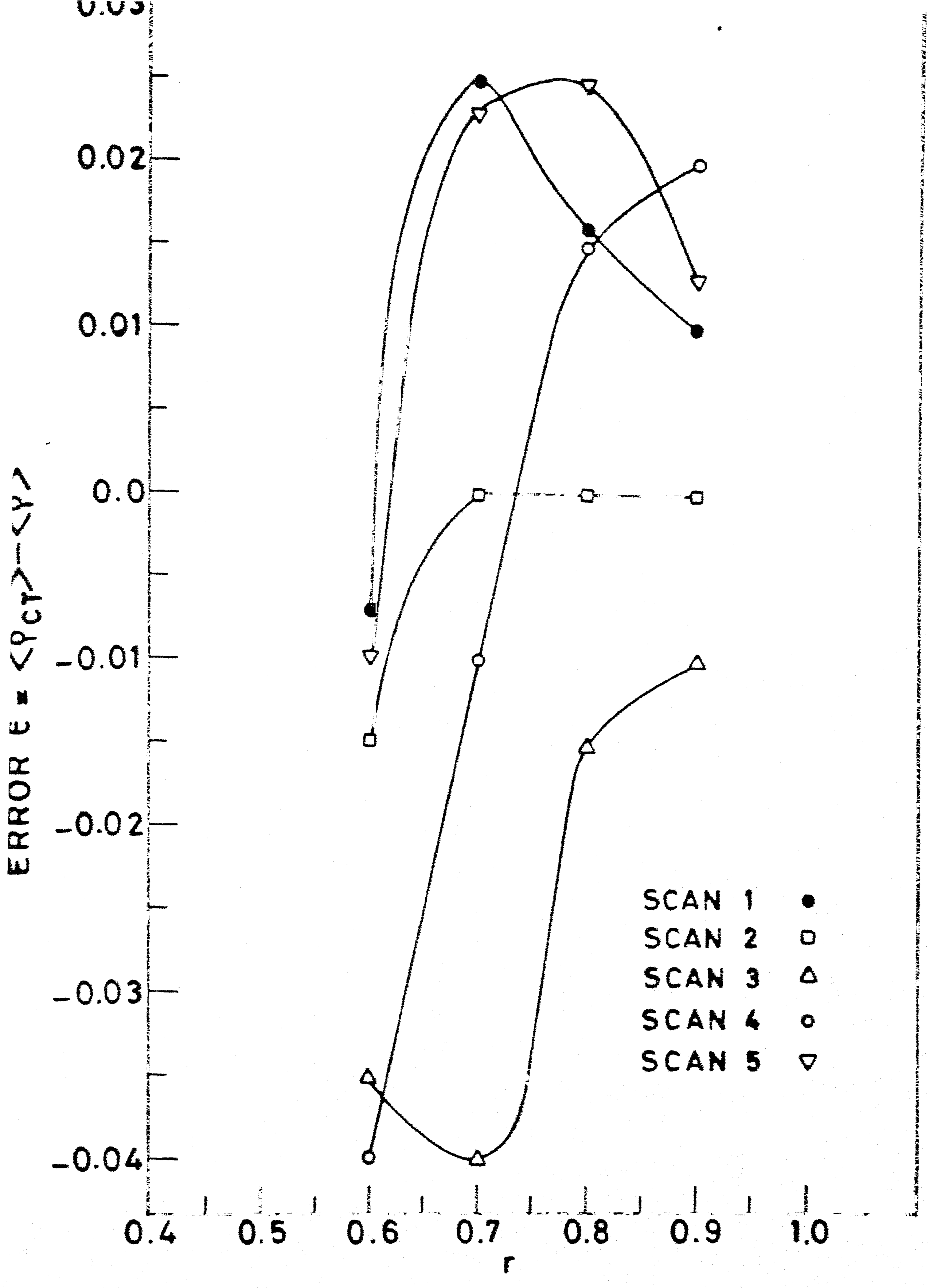
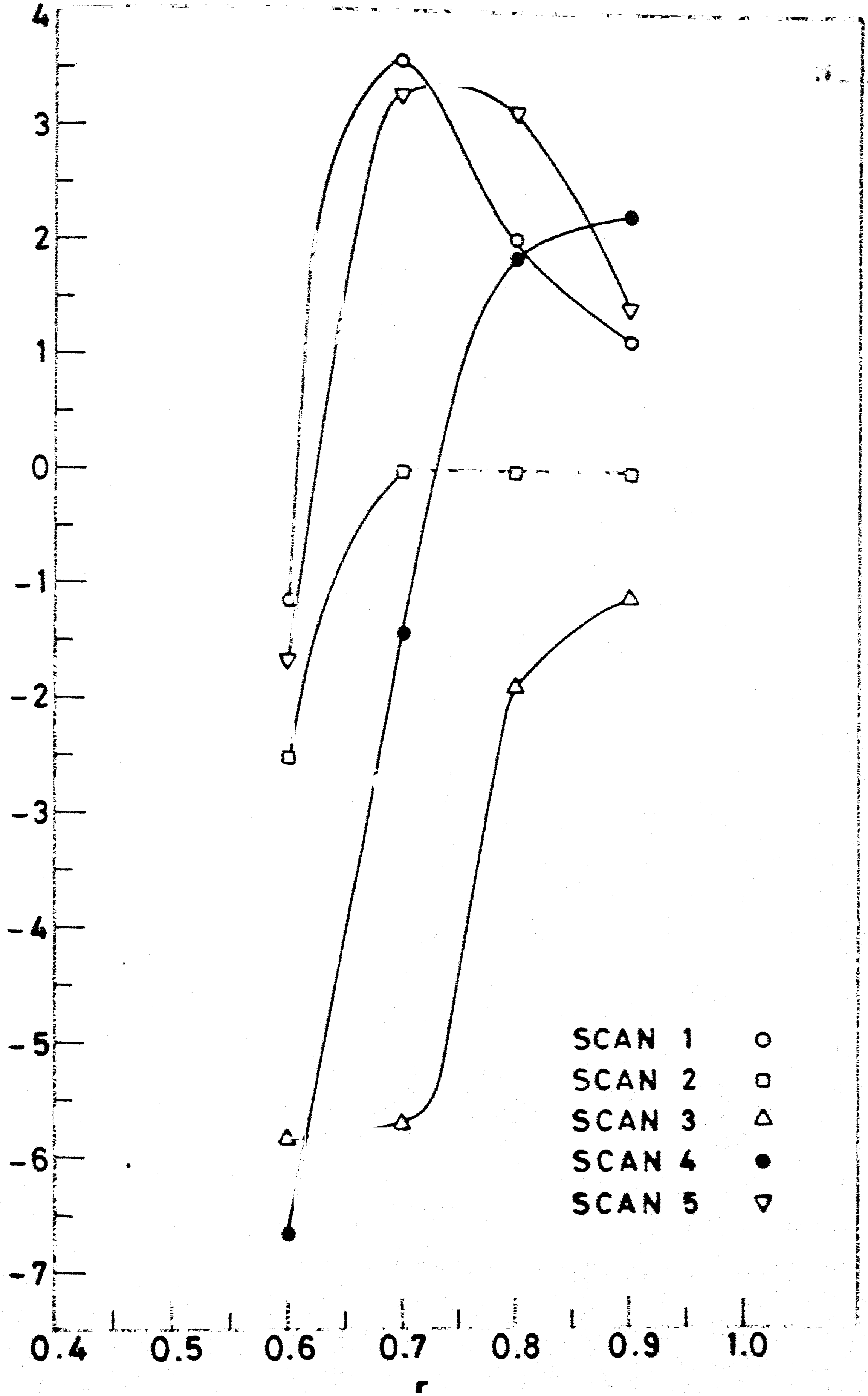


FIG.13 ERROR IN DENSITY MEASUREMENT FOR ALL SCANS.


4
3
2
1
0
-1
-2
-3
-4
-5
-6
-7
0.4
0.5
0.6
0.7
0.8
0.9
1.0
r
SCAN 1
SCAN 2
SCAN 3
SCAN 4
SCAN 5

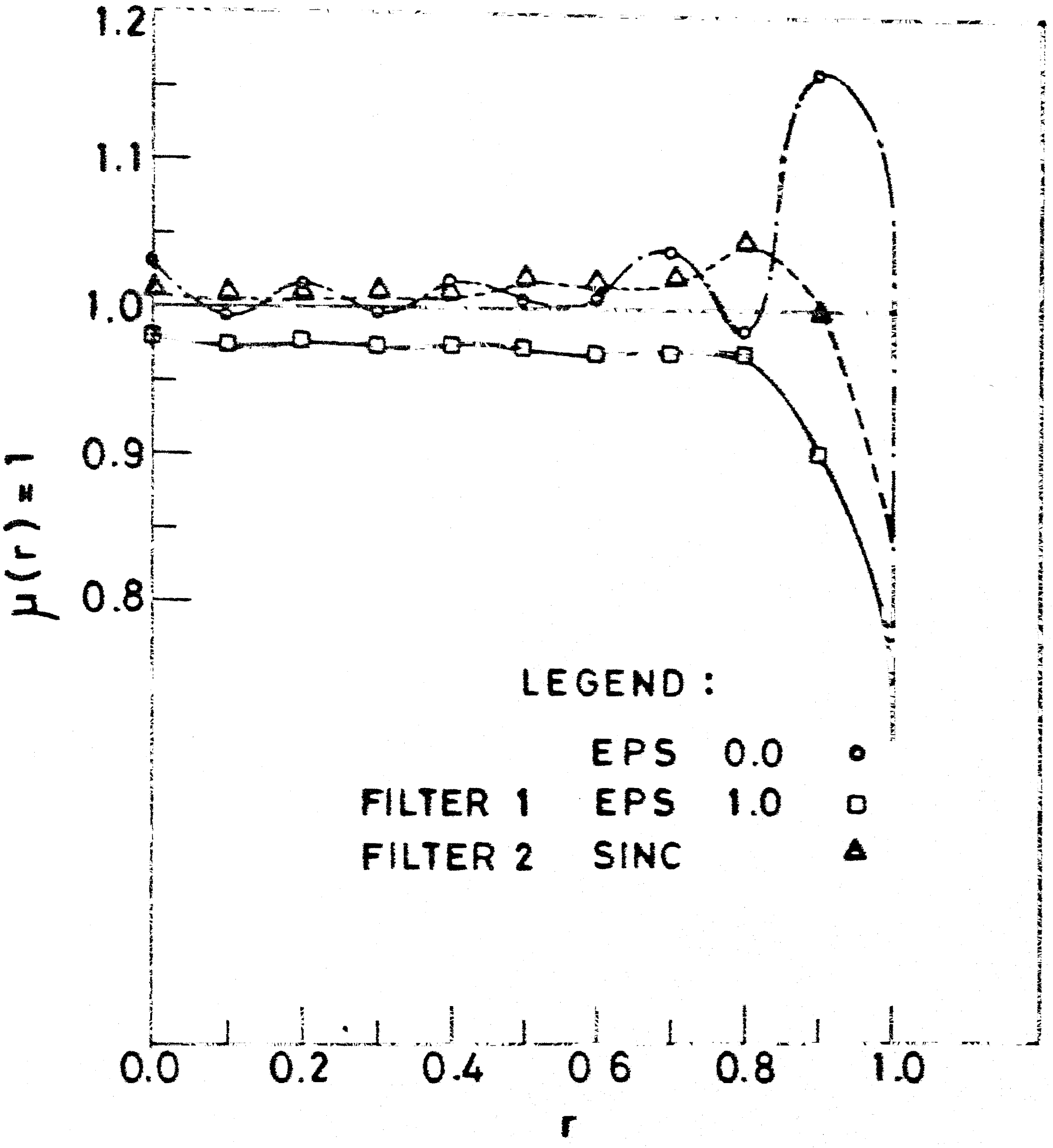


FIG.15 RESULTS WITH FILTERED VERSION.

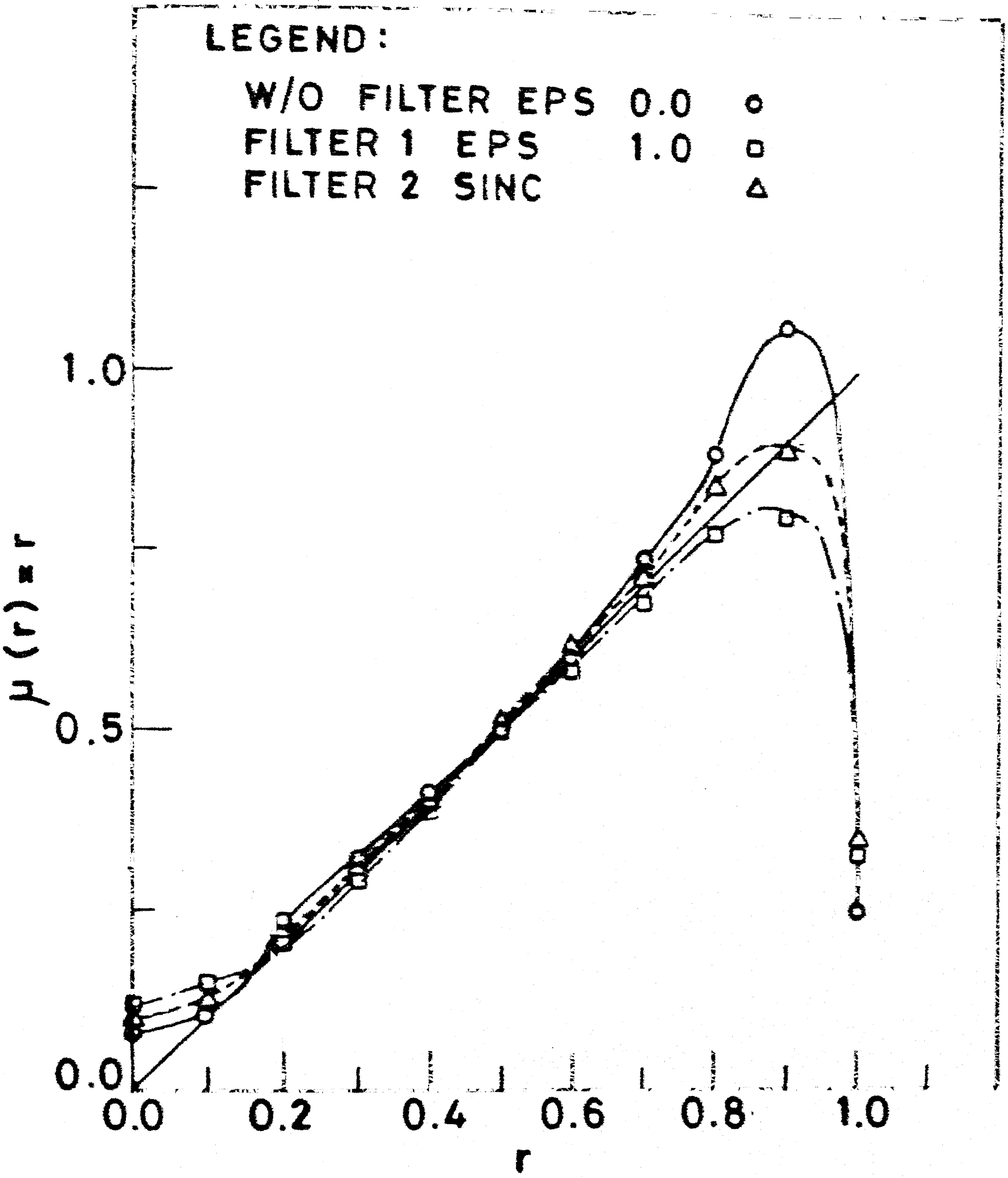


FIG. 16 RESULTS WITH FILTERED VERSION.

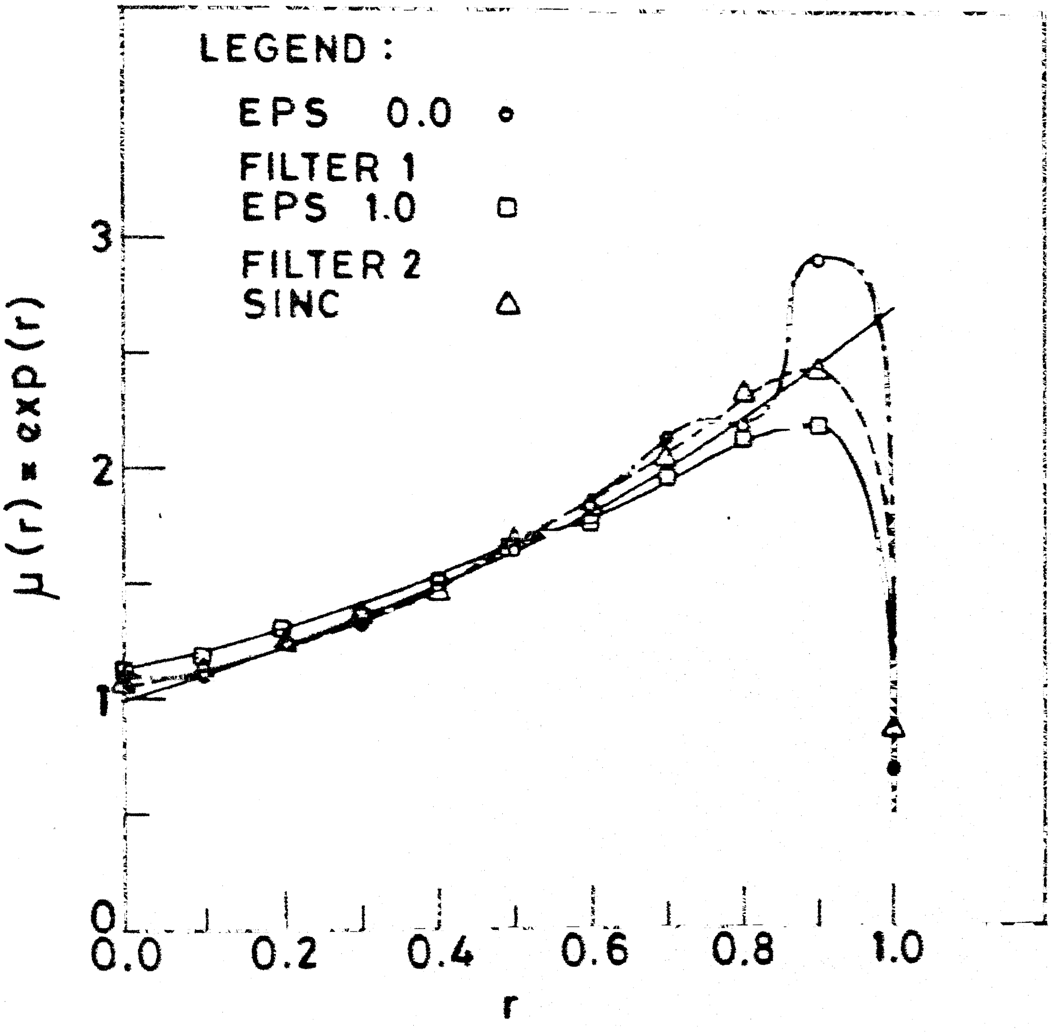


FIG.17 RESULTS WITH FILTERED VERSION.

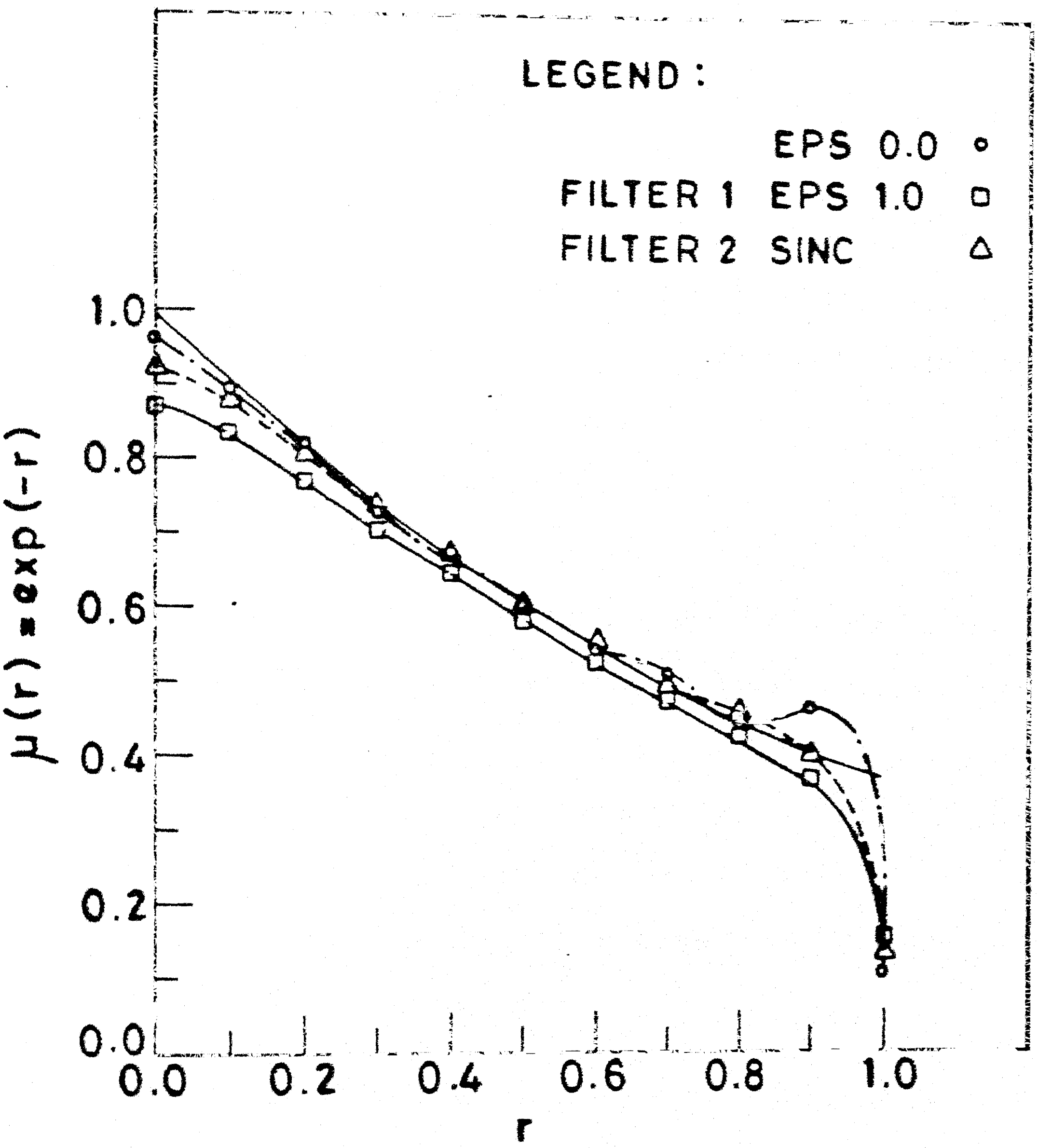


FIG.18 RESULTS WITH FILTERED VERSION.

CHAPTER 4

DISCUSSION

The proposed algorithm, using the simplified Radon Inversion Formulae, has been demonstrated to be applicable to radially symmetric two-phase flows. The unfiltered version (EPS = 0.0), incorporating a unit step cut-off in the Fourier space, gives results comparable to the more general methods of the same type [13].

However, we observe that the unfiltered version is prone to slight overshoots/undershoots, while the filtered versions are relatively free of the aforesaid oscillations. The results with the linear and sine filters show a reasonable improvement in the reconstructed values (See Figs.15-18). These filtered versions are also comparable to the general tomographic methods involving the same filter-types [13].

The bubbly flow projection data processed by this algorithm was in the range of

$$0.6 \leq \langle \rho \rangle \leq 0.9 \quad g/cm^3$$

which is equivalent to,

$$0.1 \leq \langle \alpha \rangle \leq 0.4 ,$$

where α denotes the void fraction for the air water flow systems. The values of $\langle \rho_{CT} \rangle$ are within $\pm$ 0.03 g/cm^3 of

the true values, $<\rho>$ This magnitude of the deviation compares favourably compared to the other existing methods of measuring density/void fraction in two phase flows.

We also note that the results reported here are comparable to the other algorithms developed [7,12] for radially symmetric situations.

Scan No. 1

DEGREES

TYPE SCAN	-30	-27.5	-25	-22.5	-20	-17.5	-15	-12.5	-10	-7.5	-5	-2.5	0	2.5	5	7.5	10	12.5	15	17.5	20	22.5	25	27.5	30
AIR	2869	2063	2084	2315	2404	2447	2485	2490	2516	2528	2534	2532	2527	2522	2526	2513	2492	2471	2449	2398	2308	2149	1743	2834	2842
46% VOID	2875	2109	1573	1518	1483	1448	1435	1434	1443	1456	1465	1464	1470	1469	1463	1466	1456	1423	1452	1464	1482	1541	1703	2830	2832
40% VOID	2826	2100	1552	1469	1402	1380	1351	1339	1332	1345	1349	1348	1342	1348	1361	1345	1357	1342	1364	1384	1422	1497	1667	2835	2846
30% VOID	2857	2061	1517	1377	1278	1218	1176	1160	1146	1130	1133	1126	1136	1139	1141	1158	1171	1176	1209	1269	1314	1430	1646	2835	2837
20% VOID	2864	2074	1500	1331	1218	1133	1079	1055	1018	1010	1002	1001	999	1005	1018	1023	1048	1077	1125	1198	1261	1390	1630	2824	2819
10% VOID	2879	2035	1505	1308	1173	1082	1005	958	913	894	884	876	894	894	902	923	952	991	1041	1127	1218	1370	1632	2800	2815
WATER	2885	2062	1495	1266	1106	996	913	858	811	781	758	748	740	745	765	790	826	877	951	1044	1153	1359	1663	2850	2835
WALNUT (ρ=.732)	2853	2082	1742	1565	1449	1392	1335	1265	1206	1159	1141	1121	1126	1108	1113	1132	1161	1207	1282	1365	1458	1619	1705	2846	2846
PINE (ρ=.41)	2858	2060	1773	1863	1800	1754	1705	1678	1639	1616	1601	1599	1595	1594	1610	1621	1645	1669	1715	1750	1782	1856	1710	2834	2827

Scan No. 2

DEGREES

TYPE SCAN	-30	-27.5	-25	-22.5	-20	-17.5	-15	-12.5	-10	-7.5	-5	-2.5	0	2.5	5	7.5	10	12.5	15	17.5	20	22.5	25	27.5	30
AIR													2574												2878
40% VOID	2901	2124	1563	1485	1429	1378	1359	1356	1358	1350	1374	1380	1387	1392	1389	1379	1386	1380	1387	1403	1435	1525	1749	2878	2883
30% VOID	2914	2103	1548	1411	1322	1268	1218	1203	1186	1183	1184	1181	1188	1193	1196	1203	1225	1246	1272	1322	1359	1476	1737	2862	2879
20% VOID	2907	2089	1541	1379	1266	1188	1126	1097	1062	1048	1043	1038	1040	1048	1047	1077	1090	1123	1176	1236	1310	1453	1735	2882	2874
10% VOID	2902	2090	1532	1342	1198	1108	1037	985	950	922	907	908	910	919	932	959	975	1020	1093	1166	1254	1435	1724	2883	2885
WATER													746												

Scan No. 3

DEGREES

TYPE SCAN	-30	-27.5	-25	-22.5	-20	-17.5	-15	-12.5	-10	-7.5	-5	-2.5	0	2.5	5	7.5	10	12.5	15	17.5	20	22.5	25	27.5	30
AIR													2605												2917
40% VOID	2944	2156	1595	1509	1440	1404	1349	1370	1386	1381	1399	1401	1426	1411	1424	1421	1400	1411	1432	1442	1472	1550	1767	2936	2938
30% VOID	2965	2172	1589	1447	1379	1308	1283	1257	1247	1236	1254	1256	1251	1249	126[illegible]	1276	1277	1285	1308	1358	1386	1504	1750	2910	2912
20% VOID	2949	2119	1550	1380	1253	1185	1136	1097	1072	1054	1050	1063	1077	1073	1079	1088	1128	1152	1188	1241	1330	1480	1759	2920	2917
10% VOID	2950	2120	1542	1342	1203	1110	1038	991	948	942	926	918	922	924	955	973	990	1043	1096	1170	1279	1454	1743	2917	2926
WATER													763												

Scan No. 4

DEGREES

TYPE SCAN	-30	-27.5	-25	-22.5	-20	-17.5	-15	-12.5	-10	-7.5	-5	-2.5	0	2.5	5	7.5	10	12.5	15	17.5	20	22.5	25	27.5	30
AIR													2575												2893
40% VOID	2895	2124	1574	1501	1443	1426	1405	1396	1406	1424	1413	1421	1422	1438	1414	1422	1413	1418	1410	1416	1446	1524	1728	2868	2860
30% VOID	2890	2095	1543	1416	1312	1278	1224	1206	1203	1195	1197	1200	1210	1208	1221	1209	1239	1248	1264	1307	1372	1486	1735	2857	2860
20% VOID	2884	2080	1530	1351	1234	1170	1105	1061	1041	1040	1026	1030	1034	1038	1049	1053	1087	1122	1171	1225	1303	1456	1722	2856	2853
10% VOID	2885	2075	1518	1311	1181	1083	1007	964	926	905	891	889	882	895	906	927	962	1014	1070	1151	1246	1421	1726	2860	2862
WATER													759												

Scan No. 5

DEGREES

TYPE SCAN	-30	-27.5	-25	-22.5	-20	-17.5	-15	-12.5	-10	-7.5	-5	-2.5	0	2.5	5	7.5	10	12.5	15	17.5	20	22.5	25	27.5	30
AIR													2575												2900
40% VOID	2897	2123	1566	1499	1428	1391	1355	1352	1340	1342	1361	1364	1367	1369	1379	1372	1370	1369	1402	1411	1460	1531	1739	2895	2885
30% VOID	2916	2121	1559	1413	1316	1237	1194	1174	1164	1145	1141	1143	1144	1153	1154	1169	1174	1218	1234	1290	1352	1474	1729	2866	2875
20% VOID	2909	2091	1534	1356	1227	1159	1090	1052	1026	1010	997	1001	1006	999	1027	1035	1065	1108	1159	1212	1287	1452	1729	2874	2882
10% VOID	2906	2102	1521	1308	1172	1079	1009	967	918	907	881	887	886	894	908	930	972	1011	1080	1146	1258	1423	1729	2884	2879
WATER													757												

APPENDIX B

TABLE-B1

(Results for Air, Pine, Walnut and Water)

Case	$\langle \rho \rangle$, g/cm^3	$\langle CTN \rangle$
Air	0.000	0.000
Pine	0.410	0.081
Walnut	0.732	0.146
Water	1.000	0.212

TABLE-B2

(Reconstructed Densities)

Scan No. \ $\langle \rho \rangle$ g/cm^3	0.6	0.7	0.8	0.9
1	0.59	0.72	0.82	0.91
2	0.58	0.70	0.80	0.90
3	0.57	0.67	0.78	0.89
4	0.56	0.69	0.81	0.92
5	0.59	0.72	0.82	0.91

TABLE-B3

(Reconstructed <CTNs>)

Scan No. \ $\langle \rho \rangle$ g/cm^3	0.6	0.7	0.8	0.9
1	0.1165	0.1443	0.1638	0.1829
2	0.1150	0.1383	0.1601	0.1808
3	0.1114	0.1310	0.1572	0.1792
4	0.1097	0.1370	0.1610	0.1847
5	0.1157	0.1445	0.1653	0.1843

RESULTS FOR FUN = 1

P(L) = DATA

WITHOUT FILTERED VERSION

L-VAL	P(L)	RSMALL	F(R)	INT-VAL	ERROR
0.000	2.000	0.000	1.000	1.033052	-0.033052
0.100	1.990	0.100	1.000	0.995444	0.004556
0.200	1.960	0.200	1.000	1.017349	-0.017349
0.300	1.908	0.300	1.000	0.995643	0.004357
0.400	1.833	0.400	1.000	1.017732	-0.017732
0.500	1.732	0.500	1.000	1.005409	-0.005409
0.600	1.600	0.600	1.000	1.005603	-0.005603
0.700	1.428	0.700	1.000	1.040422	-0.040422
0.800	1.200	0.800	1.000	0.988325	0.011675
0.900	0.872	0.900	1.000	1.167412	-0.167412
1.000	0.000	1.000	1.000	0.270418	0.729582

WITH LINEAR FILTER

L-VAL	P(L)	RSMALL	F(R)	INT-VAL	ERROR
0.000	2.000	0.000	1.000	0.982484	0.017515
0.100	1.990	0.100	1.000	0.978126	0.021874
0.200	1.950	0.200	1.000	0.979857	0.020143
0.300	1.908	0.300	1.000	0.978210	0.021790
0.400	1.833	0.400	1.000	0.977122	0.022878
0.500	1.732	0.500	1.000	0.977493	0.022507
0.600	1.600	0.600	1.000	0.972692	0.027308
0.700	1.428	0.700	1.000	0.973752	0.026248
0.800	1.200	0.800	1.000	0.972126	0.027874
0.900	0.872	0.900	1.000	0.906990	0.093010
1.000	0.000	1.000	1.000	0.372651	0.627349

WITH SINC FILTER

L-VAL	P(L)	RSMALL	F(R)	INT-VAL	ERROR
0.000	2.000	0.000	1.000	1.009075	-0.009075
0.100	1.990	0.100	1.000	1.004987	-0.004987
0.200	1.950	0.200	1.000	1.007308	-0.007308
0.300	1.908	0.300	1.000	1.006914	-0.006914
0.400	1.833	0.400	1.000	1.007557	-0.007557
0.500	1.732	0.500	1.000	1.010950	-0.010950
0.600	1.600	0.600	1.000	1.010897	-0.010897
0.700	1.428	0.700	1.000	1.019755	-0.019755
0.800	1.200	0.800	1.000	1.043919	-0.043919
0.900	0.872	0.900	1.000	0.996266	0.003734
1.000	0.000	1.000	1.000	0.368192	0.631808

RESULTS FOR FUN = II

P(L) = DATA

WITHOUT FILTERED VERSION

L-VAL	P(L)	RSMALL	F(R)	INT-VAL	ERROR
0.000	1.000	0.000	0.000	0.078407	-0.078407
0.100	1.025	0.100	0.100	0.103694	-0.003694
0.200	1.071	0.200	0.200	0.221228	-0.021228
0.300	1.123	0.300	0.300	0.298952	0.001048
0.400	1.167	0.400	0.400	0.419104	-0.019104
0.500	1.195	0.500	0.500	0.507586	-0.007586
0.600	1.196	0.600	0.600	0.606550	-0.006550
0.700	1.153	0.700	0.700	0.741505	-0.041505
0.800	1.044	0.800	0.800	0.789629	0.010371
0.900	0.814	0.900	0.900	1.066993	-0.166993
1.000	0.000	1.000	1.000	0.254635	0.745365

WITH LINEAR FILTER

L-VAL	P(L)	RSMALL	F(R)	INT-VAL	ERROR
0.000	1.000	0.000	0.000	0.124132	-0.124132
0.100	1.025	0.100	0.100	0.157358	-0.057358
0.200	1.071	0.200	0.200	0.236552	-0.036552
0.300	1.123	0.300	0.300	0.322136	-0.022136
0.400	1.157	0.400	0.400	0.411920	-0.011920
0.500	1.195	0.500	0.500	0.504830	-0.004830
0.600	1.196	0.600	0.600	0.593349	0.006651
0.700	1.153	0.700	0.700	0.687810	0.012190
0.800	1.044	0.800	0.800	0.779142	0.020858
0.900	0.814	0.900	0.900	0.803276	0.096724
1.000	0.000	1.000	1.000	0.338295	0.661705

WITH SINC FILTER

L-VAL	P(L)	RSMALL	F(R)	INT-VAL	ERROR
0.000	1.000	0.000	0.000	0.086652	-0.086652
0.100	1.025	0.100	0.100	0.126537	-0.026537
0.200	1.071	0.200	0.200	0.217543	-0.017543
0.300	1.123	0.300	0.300	0.313544	-0.013544
0.400	1.157	0.400	0.400	0.412496	-0.012496
0.500	1.195	0.500	0.500	0.514872	-0.014872
0.600	1.196	0.600	0.600	0.614153	-0.014153
0.700	1.153	0.700	0.700	0.722440	-0.022440
0.800	1.044	0.800	0.800	0.846338	-0.046338
0.900	0.814	0.900	0.900	0.896221	0.003779
1.000	0.000	1.000	1.000	0.343932	0.656068

RESULTS FOR FUN = EXP(r)

P(L) = DATA

WITHOUT FILTERED VERSION

L-VAL	P(L)	RSMALL	F(R)	INI-VAL	ERROR
0.000	3.437	0.000	1.000	1.138516	-0.138516
0.100	3.457	0.100	1.105	1.102060	0.003111
0.200	3.491	0.200	1.221	1.274623	-0.053220
0.300	3.519	0.300	1.350	1.343425	0.006434
0.400	3.527	0.400	1.492	1.543105	-0.051280
0.500	3.496	0.500	1.649	1.668565	-0.019844
0.600	3.404	0.600	1.822	1.840291	-0.018172
0.700	3.215	0.700	2.014	2.127485	-0.113733
0.800	2.859	0.800	2.226	2.198562	0.026979
0.900	2.219	0.900	2.460	2.915379	-0.455776
1.000	0.000	1.000	2.718	0.693351	2.024931

WITH LINEAR FILTER

L-VAL	P(L)	RSMALL	F(R)	INI-VAL	ERROR
0.000	3.437	0.000	1.000	1.130545	-0.130545
0.100	3.457	0.100	1.105	1.161727	-0.056556
0.200	3.491	0.200	1.221	1.260079	-0.038676
0.300	3.519	0.300	1.350	1.371043	-0.021184
0.400	3.527	0.400	1.492	1.500060	-0.008235
0.500	3.496	0.500	1.649	1.648885	-0.000164
0.600	3.404	0.600	1.822	1.799902	0.022217
0.700	3.215	0.700	2.014	1.983261	0.030491
0.800	2.859	0.800	2.226	2.176438	0.049103
0.900	2.219	0.900	2.460	2.206919	0.252684
1.000	0.000	1.000	2.718	0.927383	1.790899

WITH SINC FILTER

L-VAL	P(L)	RSMALL	F(R)	INI-VAL	ERROR
0.000	3.437	0.000	1.000	1.106184	-0.106184
0.100	3.457	0.100	1.105	1.144564	-0.039393
0.200	3.491	0.200	1.221	1.256031	-0.034628
0.300	3.519	0.300	1.350	1.380233	-0.030374
0.400	3.527	0.400	1.492	1.522628	-0.030804
0.500	3.496	0.500	1.649	1.688138	-0.039417
0.600	3.404	0.600	1.822	1.861260	-0.039141
0.700	3.215	0.700	2.014	2.076739	-0.062986
0.800	2.859	0.800	2.226	2.354704	-0.129163
0.900	2.219	0.900	2.460	2.453574	0.006029
1.000	0.000	1.000	2.718	0.937486	1.780796

RESULTS FOR FUN = exp(-r)

P(L) = DATA

WITHOUT FILTERED VERSION

L-VAL	P(L)	RSMALL	F(R)	INT-VAL	ERROR
0.000	1.264	0.000	1.000	0.969042	0.030958
0.100	1.234	0.100	0.905	0.896158	0.008679
0.200	1.159	0.200	0.819	0.822768	-0.004037
0.300	1.084	0.300	0.741	0.737237	0.003581
0.400	0.986	0.400	0.670	0.676513	-0.006193
0.500	0.879	0.500	0.607	0.607694	-0.001164
0.600	0.754	0.600	0.549	0.550756	-0.001944
0.700	0.64[illegible]	0.700	0.497	0.511359	-0.014774
0.800	0.504	0.800	0.449	0.444774	0.004555
0.900	0.343	0.900	0.407	0.468666	-0.062096
1.000	0.000	1.000	0.368	0.105423	0.262456

WITH LINEAR FILTER

L-VAL	P(L)	RSMALL	F(R)	INT-VAL	ERROR
0.000	1.254	0.000	1.000	0.874447	0.125553
0.100	1.234	0.100	0.905	0.839713	0.055125
0.200	1.159	0.200	0.819	0.775718	0.043012
0.300	1.084	0.300	0.741	0.708550	0.032269
0.400	0.985	0.400	0.670	0.644976	0.025344
0.500	0.879	0.500	0.607	0.586535	0.019896
0.600	0.751	0.600	0.549	0.531310	0.017501
0.700	0.64[illegible]	0.700	0.497	0.482955	0.013531
0.800	0.504	0.800	0.449	0.438275	0.011054
0.900	0.343	0.900	0.407	0.375638	0.030931
1.000	0.000	1.000	0.356	0.151080	0.216799

WITH SINC FILTER

L-VAL	P(L)	RSMALL	F(R)	INT-VAL	ERROR
0.000	1.254	0.000	1.000	0.929526	0.070474
0.100	1.234	0.100	0.905	0.888782	0.016055
0.200	1.159	0.200	0.819	0.814566	0.004155
0.300	1.084	0.300	0.741	0.739895	0.000923
0.400	0.985	0.400	0.670	0.671094	-0.000774
0.500	0.879	0.500	0.607	0.609357	-0.002826
0.600	0.754	0.600	0.549	0.552115	-0.003304
0.700	0.64[illegible]	0.700	0.497	0.503478	-0.006893
0.800	0.504	0.800	0.449	0.455285	-0.015956
0.900	0.343	0.900	0.407	0.405923	0.000647
1.000	0.000	1.000	0.368	0.144746	0.223133

FUNCTION= 1, 0,

RESULTS WITHOUT FILTER

L-VAL	DATA	RSMALL	F(R)	INT-VAL	ERROR
0.000	0.800	0.000	1.000	0.873292	0.126708
0.100	0.775	0.100	1.000	1.094587	-0.094587
0.200	0.693	0.200	1.000	0.974604	0.025396
0.300	0.529	0.300	1.000	1.188855	-0.188855
0.400	0.000	0.400	0.000	0.264220	-0.264220
0.500	0.000	0.500	0.000	-0.021588	0.021588
0.600	0.000	0.600	0.000	-0.005796	0.005796
0.700	0.000	0.700	0.000	0.012578	-0.012578
0.800	0.000	0.800	0.000	-0.012079	0.012079
0.900	0.000	0.900	0.000	0.008302	-0.008302
1.000	0.000	1.000	0.000	-0.003474	0.003474

RESULTS WITH LINEAR FILTER

L-VAL	DATA	RSMALL	F(R)	INT-VAL	ERROR
0.000	0.800	0.000	1.000	0.950976	0.049024
0.100	0.775	0.100	1.000	0.960556	0.039444
0.200	0.693	0.200	1.000	0.958434	0.041566
0.300	0.529	0.300	1.000	0.886979	0.113021
0.400	0.000	0.400	0.000	0.340691	-0.340691
0.500	0.000	0.500	0.000	0.034996	-0.034996
0.600	0.000	0.600	0.000	0.019467	-0.019467
0.700	0.000	0.700	0.000	0.007950	-0.007950
0.800	0.000	0.800	0.000	0.005277	-0.005277
0.900	0.000	0.900	0.000	0.003728	-0.003728
1.000	0.000	1.000	0.000	0.002129	-0.002129

RESULTS WITH SINC FILTER

L-VAL	DATA	RSMALL	F(R)	INT-VAL	ERROR
0.000	0.800	0.000	1.000	1.021504	-0.021504
0.100	0.775	0.100	1.000	1.032216	-0.032216
0.200	0.693	0.200	1.000	1.056784	-0.056784
0.300	0.529	0.300	1.000	0.998335	0.001665
0.400	0.000	0.400	0.000	0.350906	-0.350906
0.500	0.000	0.500	0.000	-0.002565	0.002565
0.600	0.000	0.600	0.000	0.002971	-0.002971
0.700	0.000	0.700	0.000	-0.000975	0.000975
0.800	0.000	0.800	0.000	0.000096	-0.000096
0.900	0.000	0.900	0.000	0.000230	-0.000230
1.000	0.000	1.000	0.000	-0.000278	0.000278

FUNCTION= 0, 1, 0,

RESULTS WITHOUT FILTER

L-VAL	DATA	RSMALL	F(R)	INT-VAL	ERROR
0.000	0.400	0.000	0.000	0.004273	-0.004273
0.100	0.428	0.100	0.000	-0.168250	0.168250
0.200	0.693	0.200	1.000	0.752535	0.247465
0.300	0.529	0.300	1.000	1.189902	-0.189902
0.400	0.000	0.400	0.000	0.277719	-0.277719
0.500	0.000	0.500	0.000	-0.032493	0.032493
0.600	0.000	0.600	0.000	-0.000701	0.000701
0.700	0.000	0.700	0.000	0.012591	-0.012591
0.800	0.000	0.800	0.000	-0.015140	0.015140
0.900	0.000	0.900	0.000	0.012185	-0.012185
1.000	0.000	1.000	0.000	-0.006480	0.006480

RESULTS WITH LINEAR FILTER

L-VAL	DATA	RSMALL	F(R)	INT-VAL	ERROR
0.000	0.400	0.000	0.000	-0.031784	0.031784
0.100	0.428	0.100	0.000	0.111237	-0.111237
0.200	0.693	0.200	1.000	0.662904	0.337096
0.300	0.529	0.300	1.000	0.865200	0.134800
0.400	0.000	0.400	0.000	0.329495	-0.329495
0.500	0.000	0.500	0.000	0.030962	-0.030962
0.600	0.000	0.600	0.000	0.017311	-0.017311
0.700	0.000	0.700	0.000	0.006223	-0.006223
0.800	0.000	0.800	0.000	0.004505	-0.004505
0.900	0.000	0.900	0.000	0.002997	-0.002997
1.000	0.000	1.000	0.000	0.001633	-0.001633

RESULTS WITH SINC FILTER

L-VAL	DATA	RSMALL	F(R)	INT-VAL	ERROR
0.000	0.400	0.000	0.000	-0.148578	0.148578
0.100	0.428	0.100	0.000	0.034873	-0.034873
0.200	0.693	0.200	1.000	0.734247	0.265753
0.300	0.529	0.300	1.000	1.000623	-0.000623
0.400	0.000	0.400	0.000	0.348814	-0.348814
0.500	0.000	0.500	0.000	-0.002141	0.002141
0.600	0.000	0.600	0.000	0.003125	-0.003125
0.700	0.000	0.700	0.000	-0.001235	0.001235
0.800	0.000	0.800	0.000	0.000282	-0.000282
0.900	0.000	0.900	0.000	0.000154	-0.000154
1.000	0.000	1.000	0.000	-0.000290	0.000290

DELTA -FUNCTION
WITHOUT FILTERED VERSION

L-VALUE	DATA	RSMALL	INTEGRAL VALUE
0.000	1.00	0.00	6.371062
0.100	0.00	0.10	1.042347
0.200	0.00	0.20	0.012490
0.300	0.00	0.30	-0.091482
0.400	0.00	0.40	0.079255
0.500	0.00	0.50	-0.045604
0.600	0.00	0.60	0.012038
0.700	0.00	0.70	0.012054
0.800	0.00	0.80	-0.023465
0.900	0.00	0.90	0.023228
1.000	0.00	1.00	-0.015080

DELTA -FUNCTION
RESULTS WITH LINEAR FILTER

L-VALUE	DATA	RSMALL	INTEGRAL VALUE
0.000	1.00	0.00	2.692080
0.100	0.00	0.10	1.087312
0.200	0.00	0.20	0.030145
0.300	0.00	0.30	0.026919
0.400	0.00	0.40	0.005704
0.500	0.00	0.50	0.002051
0.600	0.00	0.60	0.003769
0.700	0.00	0.70	-0.000012
0.800	0.00	0.80	0.001404
0.900	0.00	0.90	0.000579
1.000	0.00	1.00	0.000110

DELTA -FUNCTION
RESULTS WITH SINC FILTER

L-VALUE	DATA	RSMALL	INTEGRAL VALUE
0.000	1.00	0.00	3.278601
0.100	0.00	0.10	1.307053
0.200	0.00	0.20	-0.020768
0.300	0.00	0.30	0.011097
0.400	0.00	0.40	-0.002024
0.500	0.00	0.50	-0.001107
0.600	0.00	0.60	0.001695
0.700	0.00	0.70	-0.001285
0.800	0.00	0.80	0.000619
0.900	0.00	0.90	-0.000047
1.000	0.00	1.00	-0.000292

RESULTS FOR AIR

R-VAL	DATA	RSMALL	VALUE	VALINT(I)
0.000	7.934736	0.000	0.343073	0.343073
0.305	7.932808	0.305	0.765102	0.765102
0.610	7.931392	0.610	0.888887	0.888887
0.914	7.929233	0.914	0.839703	0.839703
1.216	7.920841	1.216	0.717628	0.717628
1.515	7.812378	1.515	0.690321	0.690321
1.812	7.903435	1.912	0.853521	0.853521
2.105	7.782390	2.105	1.038153	1.038153
2.394	7.744137	2.394	0.990076	0.990076
2.679	7.672758	2.679	0.962430	0.962430
2.958	7.463461	2.958	1.335429	1.335429
3.232	7.949444	3.232	1.890475	1.890475
3.500	7.952263	3.500	4.785457	4.785457

AREA AVERAGE=0.000000

RESULTS FOR WATER

R-VAL	DATA	RSMALL	VALUE	VALINT(I)
0.000	6.606650	0.000	5.502898	0.343073
0.305	6.613384	0.305	0.330819	0.765102
0.610	6.639876	0.610	0.657222	0.888887
0.914	6.672033	0.914	0.659477	0.839703
1.216	6.716595	1.216	0.528999	0.717628
1.515	6.776507	1.515	0.474123	0.690321
1.812	6.857514	1.812	0.628021	0.853521
2.105	6.950815	2.105	0.832620	1.038153
2.394	7.050123	2.394	0.780065	0.990076
2.679	7.207860	2.679	0.747661	0.962430
2.958	7.416378	2.958	1.279630	1.335429
3.232	7.955074	3.232	1.898424	1.890475
3.500	7.949797	3.500	4.779936	4.785457

AREA AVERAGE=0.212267

RESULTS FOR WALNUT

L-VAL	DATA	RSMALL	VALUE	VALINT(I)	VALACT(I)
0.000	7.026427	0.000	-0.071944	0.013646	0.085590
0.305	7.010312	0.305	0.643460	0.773350	0.129889
0.610	7.014814	0.610	0.715929	0.859481	0.143552
0.914	7.031741	0.914	0.673239	0.819705	0.146466
1.216	7.057037	1.216	0.579101	0.727867	0.148766
1.515	7.095823	1.515	0.562008	0.714355	0.151748
1.812	7.156177	1.812	0.704982	0.853478	0.148496
2.105	7.218910	2.105	0.867287	1.012520	0.145233
2.394	7.284821	2.394	0.840427	0.990897	0.150470
2.679	7.389564	2.679	0.861379	0.996381	0.135001
2.958	7.441320	2.958	1.263031	1.289660	0.026629
3.232	7.953670	3.232	1.916685	1.917099	0.000414
3.500	7.953670	3.500	4.681117	4.680028	-0.001089

AREA AVERAGE= 0.146203

RESULTS FOR PINE

L-VAL	DATA	RSMALL	VALUE	VALINT(I)	VALACT(I)
0.000	7.374629	0.000	-0.066586	0.013646	0.080231
0.305	7.374002	0.305	0.692213	0.773350	0.081136
0.610	7.383989	0.610	0.779395	0.859481	0.080087
0.914	7.390799	0.914	0.737285	0.819705	0.082420
1.216	7.405496	1.216	0.648482	0.727867	0.079384
1.515	7.419980	1.515	0.632811	0.714355	0.081545
1.812	7.447168	1.812	0.773668	0.853478	0.079809
2.105	7.467371	2.105	0.931737	1.012520	0.080783
2.394	7.485492	2.394	0.903411	0.990897	0.087486
2.679	7.526179	2.679	0.927431	0.996381	0.068950
2.958	7.444249	2.958	1.273474	1.289660	0.016186
3.232	7.949444	3.232	1.916761	1.917099	0.000338
3.500	7.946971	3.500	4.677223	4.680028	0.002805

AREA AVERAGE= 0.080758

L-VAL	P(L)	RSMALL	VALUE	VALINT(I)	VALACT(I)
0.000	6.795706	0.000	-0.153378	0.013645	0.167024
0.305	6.795706	0.305	0.606432	0.773350	0.164918
0.610	6.804515	0.610	0.684832	0.859481	0.174649
0.914	6.827529	0.914	0.644723	0.819705	0.174962
1.216	6.858565	1.216	0.553522	0.727857	0.174345
1.515	6.896715	1.515	0.538825	0.714355	0.175531
1.812	6.947937	1.812	0.667990	0.853473	0.185487
2.105	7.027315	2.105	0.830691	1.012520	0.181829
2.394	7.104955	2.394	0.800537	0.990897	0.190360
2.679	7.222556	2.679	0.796582	0.995381	0.199799
2.958	7.397552	2.958	1.240545	1.289550	0.049115
3.232	7.937375	3.232	1.909778	1.917099	0.007321
3.500	7.942718	3.500	4.674545	4.680028	0.005483

L-VAL	P(L)	RSMALL	VALUE	VALINT(I)	VALACT(I)
0.000	6.905755	0.000	-0.151730	0.013646	0.165376
0.305	6.912743	0.305	0.628167	0.773350	0.145183
0.610	6.925595	0.610	0.718409	0.859481	0.141072
0.914	6.930495	0.914	0.668358	0.819705	0.151348
1.216	6.954639	1.216	0.576558	0.727857	0.151309
1.515	6.981935	1.515	0.557475	0.714355	0.156881
1.812	7.025538	1.812	0.689333	0.853478	0.164144
2.105	7.038409	2.105	0.851948	1.012520	0.160572
2.394	7.139560	2.394	0.814831	0.990897	0.176067
2.679	7.237059	2.679	0.804410	0.995381	0.191971
2.958	7.396335	2.958	1.238869	1.289550	0.050791
3.232	7.945910	3.232	1.912566	1.917099	0.004533
3.500	7.941138	3.500	4.675837	4.680028	0.004191

L-VAL	P(L)	RSMALL	VALUE	VALINT(I)	VALACT(I)

```
C ******************************************************************
C
C THIS PROGRAM EVALUATES THE FUNCTION f(r) FOR DIFFERENT
C VALUES OF r BETWEEN 0.0 - 1.0 IN STEP OF 0.10
C INPUT DATA REQUIRED
C NDATA values of 1 and n(1) FOR THE FUNCTIONS LIKE
C F(R)=1.0,r,exp(r),exp(-r) and for different flow regiems
C ARE GIVEN AS INPUT DATA FOR THIS PROGRAM
C
C ******************************************************************
      COMMON/I1/NDATA,p(20),plval(20)
      COMMON/I6/EPSWR,RC,INDWR,IWRTYP,IAREA
      COMMON/I5/VALACT(15),R(15)
      DIMENSION PHATR(165),VALINT(15)
C
      PI=4.0*ATAN(1.0)
C
C NDATA  - # OF DATA POINTS
C INDWR  -=1 NO WINDOW FN IS USED
C        ->1 OR <1 WINDOW FN IS USED
C IWRTYP -=2 SINC WINDOW USED
C        ->2 OR <2 LINEAR WINDOW USED
C IAREA  -GE 0 AREA CAL IS DONE & DATA IS STORED
C        -< 0 NO AREA CAL IS DONE
C IAIR   -=1 AIR DATA READ & BACKGROUND DATA/COUNTS STORED(AIR)
C        -<1 OR >1 AREA CAL DONE
C
C READ INPUT DATA
      ACCEPT *,NDATA,INDWR,EPSWR,IWRTYP
      ACCEPT *,IAREA,IAIR
C
      IF(IAREA.LT.0)GO TO 30
      D=7.0
      DO 10 I=1,NDATA
         READ(24,*)P(I),PLVAL(I)
      P(I)=ALOG(P(I))
   10 PLVAL(I)=D*SIN(PLVAL(I)*PI/180.0)
      IF(IAIR.EQ.1)GO TO 50
      DO 20 I=1,NDATA
   20 READ(23,*)R(I),VALINT(I)
      GO TO 50
C
   30 DO 40 I=1,NDATA
   40 READ(24,*)P(I),PLVAL(I)
C SET VALUES OF WINDOW FN
   50 RC=6.0
      N=160
      IF(IAREA.LT.0)GO TO 60
      WRITE(22,150)
      GO TO 90
   60    WRITE(22,70)N
```

```
 70     FORMAT(//20X,'N-VALUE=  ',I6/)
        WRITE(22,80)
 80     FORMAT(3X,'L-VAL',5X,'P(L)',5X,'RSMALL',5X,'F(R)',
        18X,'INT-VAL',5X,'ERROR'/)
C LIMITS OF THE OUTERMOST INT'GL A & B , STEP LENGTH H
 90     A=0.0
        B=6.0
        H=(B-A)/N
        NN=N+1
C STORE PHATR FOR DIFFERENT STEPLENGTHS
        CALL PR(PHATR,NN,H)
        RSMALL=0.0
        DO 160 I=1,NDATA
        CONST=2.0*PI*RSMALL
C INITIALISE I VALUE  FOR A PARTICULAR r
        VALUE=0.0
C INTEGRATION BY SIMPSON'S 1/3 RULE BEGINS
C S17EAF IS A SYSTEM FUNCTION SUBPROGRAM WHICH EVALUATES
C BESSEL'S FUNCTION AT A GIVEN POINT
C
        DO 100 J=2,N-2,2
        RI=A+H*FLOAT(J)
        CONJ=CONST*RI
        Y=S17AEF(CONJ,IFAIL)
        VALUE=VALUE+2.0*RI*PHATR(J+1)*Y*W(RI)
 100    CONTINUE
C
        DO 110 J=1,N-1,2
        RI=A+H*FLOAT(J)
        CONJ=CONST*RI
        Y=S17AEF(CONJ,IFAIL)
 110    VALUE=VALUE+4.0*RI*PHATR(J+1)*Y*W(RI)
C
        RI=A
        CONJ=CONST*RI
        Y=S17AEF(CONJ,IFAIL)
        VALUE=VALUE+PHATR(1)*RI*Y*W(RI)
C
        RI=B
        CONJ=CONST*RI
        Y=S17AEF(CONJ,IFAIL)
        VALUE=VALUE+PHATR(NN)*RI*Y*W(RI)
        VALUE=VALUE*H/3.0
C f(R)=2.0*PI*VALUE
        VALUE=VALUE*2.0*PI
        PL=P(I)
C EXACT VALUE OF THE FUNCTION FRR=f(r)
        FRR=1.0
        IF(RSMALL.GT.0.4)FRR=0.0
        IF(IAIR.EQ.1)FRR=1.0
C
```

```
C FOR VARIOUS TEST FNS REPLACE ERR IN THE ABOVE LINE BY
C THE CORRESPONDING FUNCTIONS
C ERR = 1.0    f(r)=1.0
C ERR = RSMALL    f(r)=r  ramp fn
C ERR = EXP(RSMALL)   f(r)=exp(r) and
C ERR = EXP(-RSMALL)   f(r)=exp(-r)
C
C       IF(RSMALL.LT.0.2.OR.RSMALL.GT.0.4)ERR=0.0
C ESTIMATE THE ERROR (EXACT-RECONSTRUTED)
  120   ERR=ERR-VALUE
C
        IF(IAREA.LT.0)GO TO 130
        VALACT(I)=-(VALUE-VALINT(I))
        WRITE(22,170)PLVAL(I),P(I),RSMALL,VALUE,VALINT(I),VALACT(I)
        IF(IAIR.EQ.1)WRITE(23,*)PLVAL(I),VALUE
        RSMALL=PLVAL(I+1)
        GO TO 160
  130   WRITE(22,140),PLVAL(I),PL,RSMALL,ERR,VALUE,EPR
  140   FORMAT(2X,4(F6.3,4X),2F10.6)
        RSMALL=RSMALL+0.1
  150   FORMAT(3X,'L-VAL',9X,'P(L)',9X,'RSMALL',9X,'VALUE',
       18X,'VALINT(I)',9X,'VALACT(I)'/)
  160   CONTINUE
C
  170   FORMAT(5X,F6.3,5X,F10.6,5X,F6.3,5X,3(F10.6,5X))
        IF(IAREA.LT.0)GO TO 180
        CALL AREA
  180   CONTINUE
        STOP
        END
C
C ***************************************************************
C THIS SUBROUTINE CALCULATES PHATR VALUES FOR DIFFERENT
C STEP-LENGTHS (R - VALUES) WHICH ARE USED IN THE MAIN
C PROGRAM FOR INTEGRATION
C ***************************************************************
C
        SUBROUTINE PR(PHATR,N,H)
C
        COMMON/I2/R2PI
        COMMON/I1/NDATA,P(20),PLVAL(20)
        COMMON/I6/EPSWR,RC,INDWR,IWRTYP,IAREA
        DIMENSION PHATR(N)
        PI=4.0*ATAN(1.0)
        M=100
        IF(IAREA.GE.0)M=160
C LIMITS OF THE INTEGRAL PHAT(R)
        A=0.0
        B=1.0
        IF(IAREA.GE.0)B=3.5
        HPRIM=(B-A)/M
```

```
      R2PI=0.0
      DO 30 I=1,N
      VALUE=0.0
      DO 10 J=2,M-2,2
      SLV=A+HPRIM*FLOAT(J)
      VALUE=VALUE+2.0*PSLI(SLV)
10    CONTINUE
      DO 20 J=1,M-1,2
      SLV=A+HPRIM*FLOAT(J)
20    VALUE=VALUE+4.0*PSLI(SLV)
      SLV=A
      VALUE=VALUE+PSLI(SLV)
      SLV=B
      VALUE=VALUE+PSLI(SLV)
      VALUE=VALUE*HPRIM/3.0
      PHATR(I)=2.0*VALUE
      R=H*FLOAT(I)
      R2PI=R*2.0*PI
30    CONTINUE
      RETURN
      END
C
C *****************************************************************
C THIS SUBROUTINE INTERPOLATES THE DATA P(1) FOR A GIVEN VALUE OF 1
C
C *****************************************************************

      FUNCTION PSLI(SLI)
C
      COMMON /I1/NDATA,P(20),PLVAL(20)
      COMMON/I2/R2PI
      COMMON I
C
      IF( SLI.EQ.0.0)GO TO 30
      DO 10 I=1,NDATA
      IF(SLI.LE.PLVAL(I))GO TO 20
10    CONTINUE
20    LMAX=I
      LMIN=LMAX-1
      A=PLVAL(LMIN)
      B=PLVAL(LMAX)
      FCAL=(SLI-A)*(P(LMAX)-P(LMIN))/(B-A)
      PSLI=P(LMIN)+FCAL
      PSLI=PSLI*COS(R2PI*SLI)
      RETURN
30    PSLI=P(1)
      PSLI=PSLI*COS(R2PI*SLI)
      RETURN
      END
C
C *****************************************************************
```

```
C THIS FUNCTION SUBPROGRAM RETURNS THE WINDOW FUNCTION VALUE
C FOR A GIVEN R VALUE
C THIS HAS TWO WINDOW FUNSTIONS
C        I) LINEAR AND II) SINC
C IF INDWR EQ 1, W RETURNS 1.0 ie W(R)=1.0 ( NO WINDOW FN USED)
C IF IWRTYP EQ 2,W(R) USES SINC FN ,OTHERWISE USES LINEAR FN
C ********************************************************************
C
        FUNCTION W(R)
C
        COMMON/I6/EPSWR,RC,INDWR,IWRTYP,IAREA
C
C WHEN W(R) IS NOT USED SET W(R)=1.0
        IF(INDWR.EQ.1)GO TO 20
        IF(IWRTYP.EQ.2)GO TO 30
        IF(R.EQ.0.0)GO TO 10
C
        W=1.0-EPSWR*ABS(R)/RC
   10   RETURN
   20   W=1.0
        RETURN
   30   IF(ABS(R).LE.0.1E-5)GO TO 20
        PI=4.0*ATAN(1.0)
        W=SIN(PI*ABS(R)/RC)/(PI*ABS(R)/RC)
        RETURN
        END
C
C ********************************************************************
C THIS SUBROUTINE CALCULATES THE AVERAGE DENSITY FOR VARIOUS
C TYPES OF FLOWS
C ********************************************************************
C
        SUBROUTINE AREA
C
        COMMON/I5/VALACT(15),R(15)
        AREA=0.0
C
        DO 40 I=1,10
        VALUE=0.0
        NITRN=100
        A=0.0
        B=R(I)
        H=(B-A)/NITRN
        IF(A.EQ.B)GO TO 30
        DO 10 J=2,NITRN-2,2
        RS=A+H*FLOAT(J)
   10   VALUE=VALUE+RS*FRS(RS)*2.0
        DO 20 J=1,NITRN-1,2
        RS=A+H*FLOAT(J)
   20   VALUE=VALUE+RS*FRS(RS)*4.0
        VALUE=VALUE+A*FRS(A)
```

```
      VALUE=(VALUE+B*FRS(B))*H/3.0
      AREA=2.0*VALUE/(B*B)
 30   CONTINUE
 40   CONTINUE
      WRITE(22,50)AREA,R(10)
 50   FORMAT(//20X,'AREA AVERAGE=',F10.6,6X,'LIMITS 0.0-',F10.6)
      RETURN
      END
C
C ****************************************************************
C THIS IS AN INTERPOLATION FUNCTION SUBPROGRAM USED IN AREA
C CALULATIONS
C ****************************************************************
C
      FUNCTION FRS(X)
C
      COMMON/I5/VALACT(15),R(15)
      IF(X.EQ.0.0)GO TO 30
      DO 10 I=1,10
      IF(X.LE.R(I))GO TO 20
 10   CONTINUE
 20   LMAX=I
      LMIN=I-1
      A=R(LMIN)
      B=R(LMAX)
      FCAL=(X-A)*(VALACT(LMAX)-VALACT(LMIN))/(B-A)
      FRS=VALACT(LMIN)+FCAL
      RETURN
 30   FRS=VALACT(1)
      RETURN
      END
```

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